Interconnections between $T\bar{T}$-like flows, nonlinear electrodynamics, and modified gravity theories

MPI July 2024

MATRIX - Creswick

Roberto Tateo

Istituto Nazionale di Fisica Nucleare
INFN TORINO
Sezione di Torino

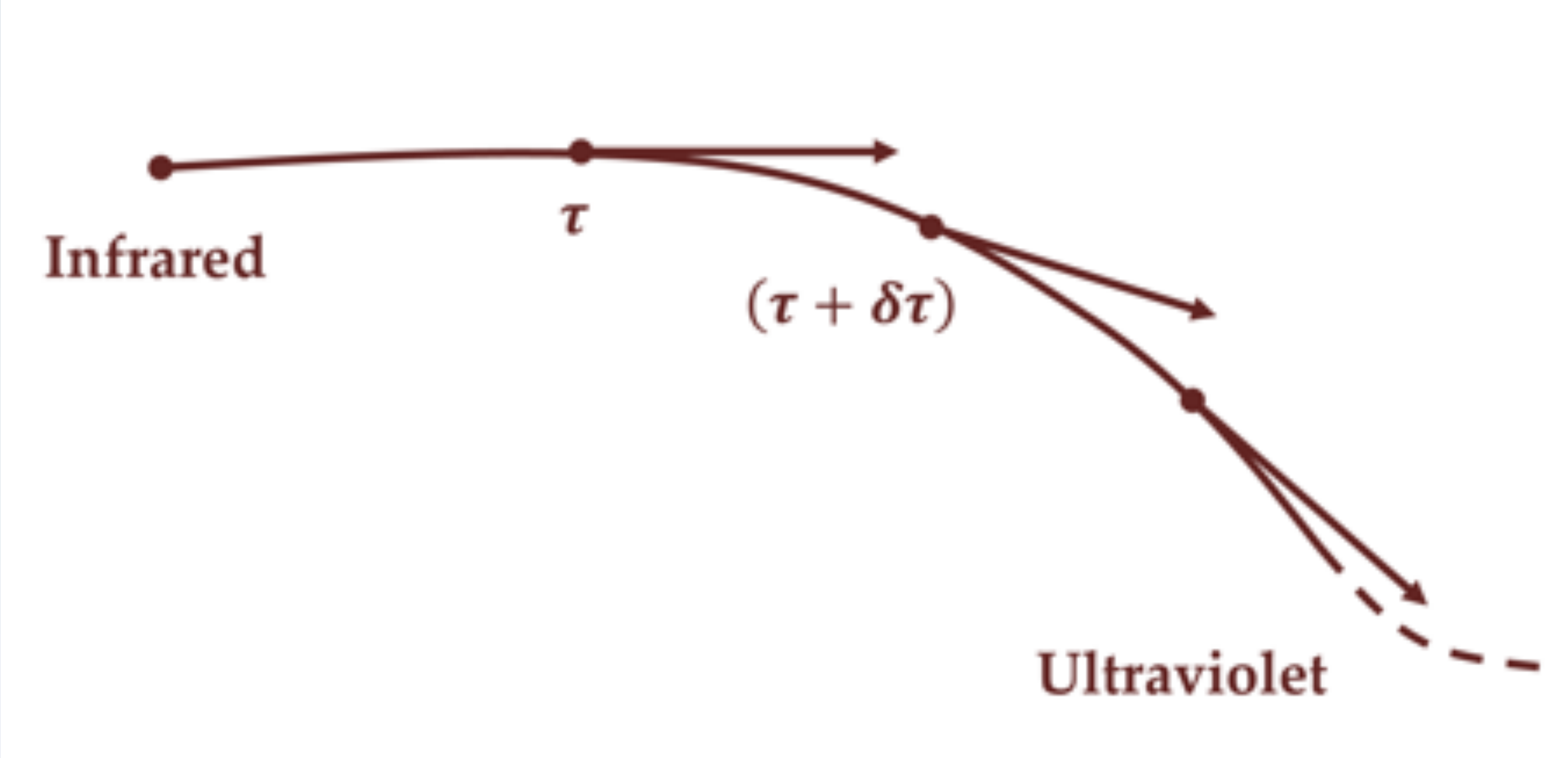
UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

Mainly based on work with:

R. Conti, J. Romano, C. Ferko, N. Brizio, J. Hou, T. Morone, S. Negro, G. Tartaglino-Mazzucchelli, H. Babaei-Aghbolagh, S. He, H. Ouyang

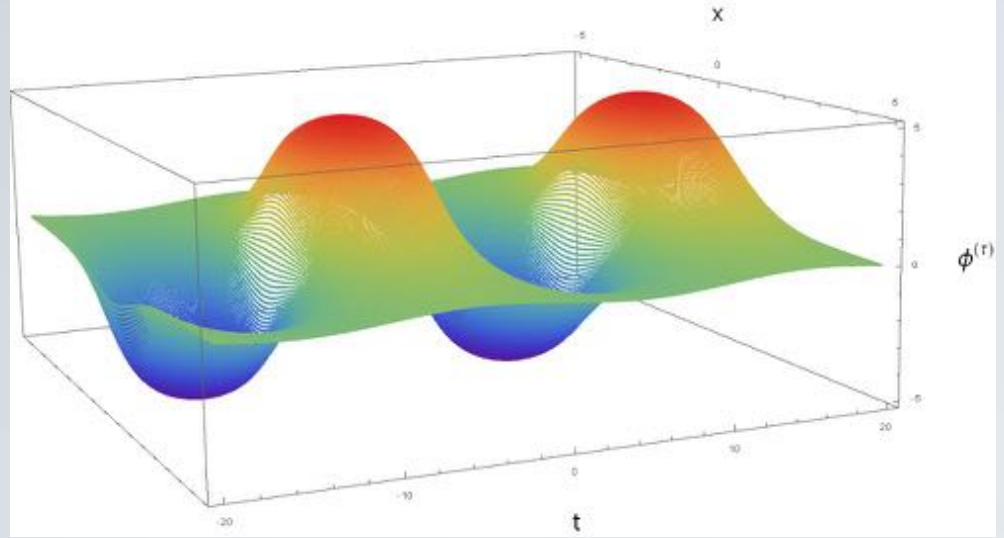
The $T\bar{T}$ Lagrangian flow equation in 2d is:

$$\begin{cases} \partial_\tau \mathscr{L}(\tau) = \det(T_{\mu\nu}(\tau))\,, \\ T_{\mu\nu}(\tau) = \dfrac{-2}{\sqrt{g}}\dfrac{\partial \mathscr{L}(\tau)}{\partial g^{\mu\nu}}\,, \end{cases}$$

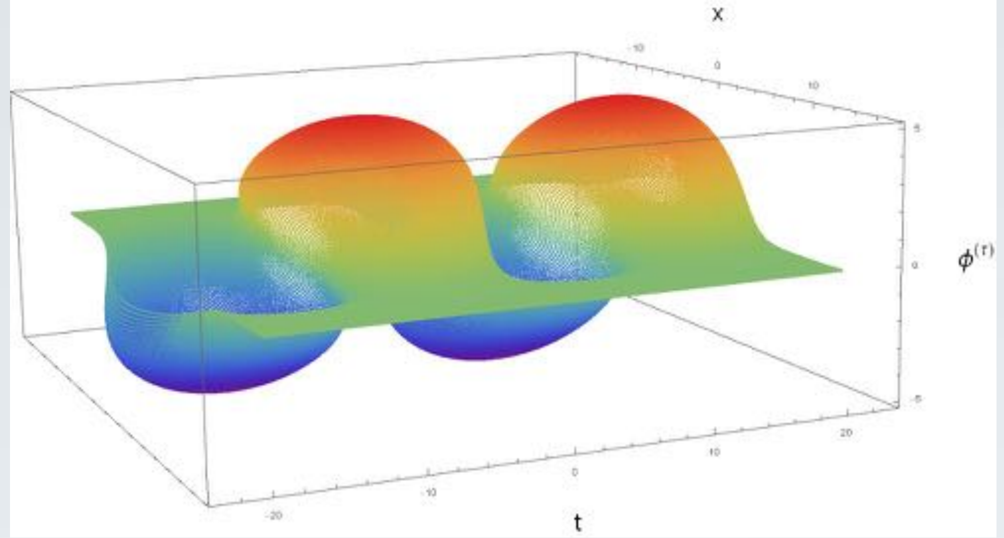


Dynamical change of coordinates = $T\bar{T}$ deformations

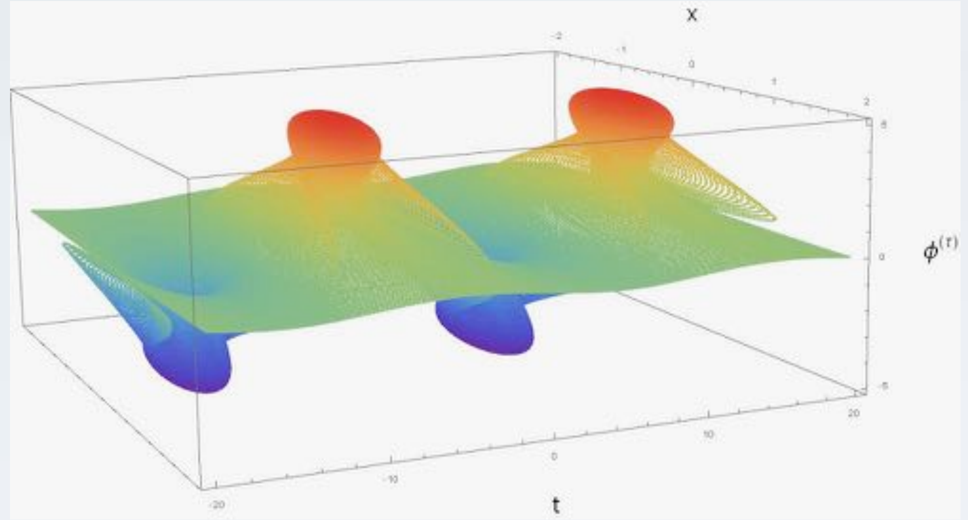
$$\mathcal{J}^{-1} = \begin{pmatrix} \partial_w z & \partial_w \bar{z} \\ \partial_{\bar{w}} z & \partial_{\bar{w}} \bar{z} \end{pmatrix} = \begin{pmatrix} 1 - \tau\Theta(\mathbf{w}) & -\tau\bar{T}(\mathbf{w}) \\ -\tau T(\mathbf{w}) & 1 - \tau\Theta(\mathbf{w}) \end{pmatrix}$$

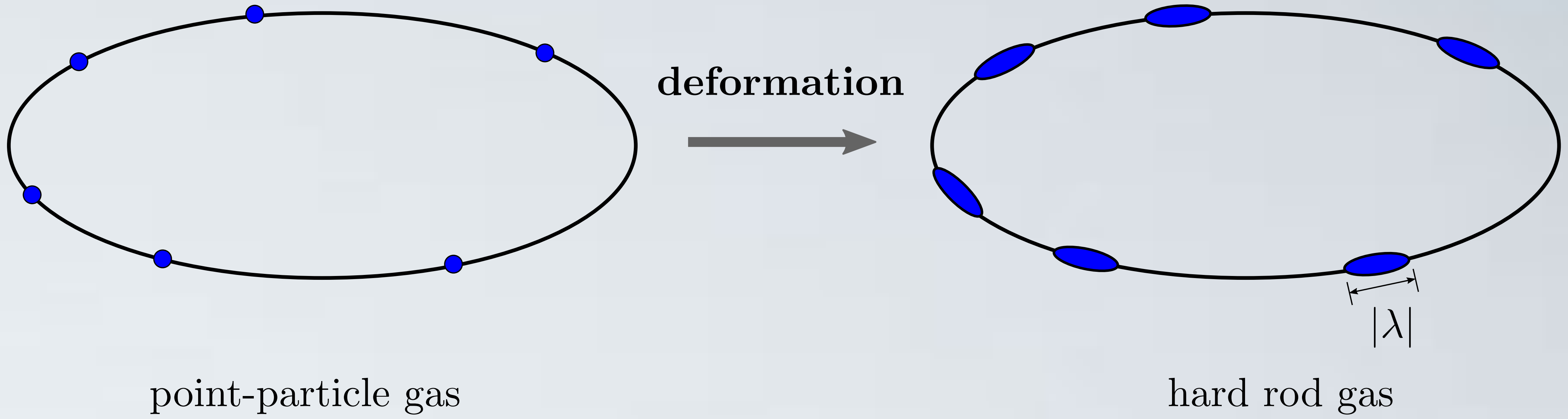


$\tau > 0$



$\tau < 0$





[Picture by Y. Jiang]

$$\phi^{(\tau)}(\mathbf{z}) = \phi^{(0)}\left(\mathbf{w}(\mathbf{z})\right) \;, \quad \mathbf{z} = (z, \bar{z}) \,, \quad \mathbf{w} = (w, \bar{w})$$

Metric tensor:

$$\boxed{\mathbf{g}'_{\mu\nu} = \delta_{\mu\nu} - \tau \epsilon_{\mu\rho} \epsilon^{\sigma}{}_{\nu} \left(2T + \tau T^2\right)^{\rho}{}_{\sigma}}$$

Proposed $T\bar{T}$ geometric intrepretations

1) There exists a random geometry interpretation of the $T\bar{T}$ deformation of quantum field theory [Cardy]

$$e^{2\delta t \int_{\mathcal{D}} \epsilon_{ik}\epsilon_{jl} T^{ij} T^{kl} d^2x} \propto \int [dh] e^{-(1/8\delta t)\int \int_{\mathcal{D}} \epsilon^{ik}\epsilon^{jl} h_{ij} h_{kl} d^2x + \int_{\mathcal{D}} h_{ij} T^{ij} d^2x}$$

(Hubbard-Stratonovich transformation)

2) Any $T\bar{T}$-deformed field theory is dynamically equivalent to its associated unperturbed theory coupled to (flat) Jackiw-Teitelboim gravity [Dubovsky-Gorbenko-Mirbabayi].

$$S_{\mathrm{M},\tau} \simeq S_{\mathrm{M}} + \int \mathrm{d}^2\mathbf{x}\sqrt{-g}\,(\varphi R - \Lambda_2) \qquad \tau \propto \Lambda_2^{-1}$$

3) The $T\bar{T}$ deformation of a generic field theory is equivalent to coupling the undeformed field theory to 2D 'ghost-free massive gravity' [Tolley].

$$S_{T\bar{T}}[\varphi, f, e] = \int \mathrm{d}^2x \, \frac{1}{2\lambda}\epsilon^{\mu\nu}\epsilon_{ab}(e^a_\mu - f^a_\mu)(e^b_\nu - f^b_\nu) + S_0[\varphi, e] \qquad \lambda \propto \tau$$

$T\bar{T}$-type perturbations in higher space-time dimensions

[M.Taylor, J.Cardy…]

$$\mathcal{L}^{\mathrm{MBI}}(\mathcal{A},\tau) = \frac{-1+\sqrt{1-\tau\,\mathrm{Tr}\left[F^2\right]+\frac{\tau^2}{4}\left(\mathrm{Tr}[F\widetilde{F}]\right)^2}}{2\tau}$$

Born-Infeld nonlinear electrodynamics 4d

[Conti-Iannella-Negro-RT]

$$\widetilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

$$\partial_\tau\mathcal{L}^{\mathrm{MBI}} = \sqrt{\det[T^{\mathrm{MBI}}]} = \frac{1}{4}\left(\frac{1}{2}\mathrm{Tr}\left[T^{\mathrm{MBI}}\right]^2 - \mathrm{Tr}\left[\left(T^{\mathrm{MBI}}\right)^2\right]\right)$$

Main objective:

Study Irrelevant/Marginal perturbations and their relation with Modified Gravity Models in d>2 :

$$S_{\mathrm{M},\tau} + S_{\mathrm{G}} \simeq S_{\mathrm{M}} + S_{\mathrm{G},\tau} \quad ?$$

$T\bar{T}$ **-type deformations in d $>$ 2**

Consider the family of deformations defined by the flow equation

$$\begin{cases} \dfrac{\partial S_{\mathrm{M},\tau}}{\partial \tau} = \displaystyle\int d^d\mathbf{x}\sqrt{-g}\,\mathcal{O}^{[a,b]}_{d,\tau}\,, \quad S_{\mathrm{M},\tau_0} := S_{\mathrm{M}} \\[2em] T_{\tau,\mu\nu} = \dfrac{-2}{\sqrt{g}}\dfrac{\delta S_{\mathrm{M},\tau}}{\delta g^{\mu\nu}} \end{cases}$$

with perturbing operator

$$\mathcal{O}^{[a,b]}_{d,\tau} := \frac{1}{d}\left(a\mathrm{tr}\left[\mathbf{T}_\tau\right]^2 - b\,\mathrm{tr}\left[\mathbf{T}^2_\tau\right]\right), \quad a, b \in \mathbb{R}\,,\ d \geq 2$$

where $\mathbf{T}_\tau = \left(g^{\mu\alpha}T_{\tau,\alpha\nu}\right)_{\mu,\nu\in\{0,\ldots,d-1\}}$ is a $d \times d$ dimensional matrix

In two dimensions, since

$$\mathcal{O}^{[1,1]}_{2,\tau} = \frac{1}{2}\left(\mathrm{tr}\,[\mathbf{T}_\tau]^2 - \,\mathrm{tr}\left[\mathbf{T}^2_\tau\right]\right) = \det[\mathbf{T}_\tau]$$

we recover the usual definition of $\mathrm{T}\overline{\mathrm{T}}$ deformations when setting $a = 1$, $b = 1$.

The metric approach

$\mathrm{T}\overline{\mathrm{T}}$-type **perturbations** can be understood as the generators of the metric flow

$$\begin{cases} \dfrac{dg_{\tau,\mu\nu}}{d\tau} = -\dfrac{4}{d}\widehat{T}_{\tau,\mu\nu} \\ \dfrac{d\widehat{T}_{\tau,\mu\nu}}{d\tau} = \dfrac{4}{d}\widehat{T}^{\alpha}_{\tau,\mu}\widehat{T}_{\tau,\alpha\nu} + \xi_\tau \widehat{T}_{\tau,\mu\nu} + \chi_\tau g_{\tau,\mu\nu} \end{cases}$$

where

$$\mathcal{O}^{[a,b]}_{d,\tau} = \frac{1}{d}\widehat{T}_{\tau,\mu\nu}T^{\mu\nu}_{\tau}$$

$$\widehat{T}_{\tau,\mu\nu} := a\,\mathrm{tr}\left[\mathbf{T}_\tau\right] g_{\tau,\mu\nu} - b\,T_{\tau,\mu\nu}$$

and where ξ_τ and χ_τ are scalar functions of the energy-momentum tensor defined as

$$\xi_\tau = \frac{2}{d}(b - da)\,\mathrm{tr}\left[\mathbf{T}_\tau\right], \qquad \chi_\tau = \frac{da - b}{d}\left(a\,\mathrm{tr}\left[\mathbf{T}_\tau\right]^2 - b\,\mathrm{tr}\left[\mathbf{T}_\tau^2\right]\right).$$

The idea is then to Taylor expand the metric $g_{\tau,\mu\nu}$ around $\tau = \tau_0$ as

$$g_{\tau,\mu\nu} = \sum_{n=0}^{\infty} \frac{g^{(n)}_{\tau_0,\mu\nu}}{n!}\left(\tau - \tau_0\right)^n, \qquad g^{(0)}_{\tau_0,\mu\nu} = g_{\mu\nu}.$$

The first two coefficients $g^{(1)}_{\tau_0,\mu\nu}$ and $g^{(2)}_{\tau_0,\mu\nu}$ are:

$$g^{(1)}_{\tau_0,\mu\nu} = -\frac{4}{d}\widehat{T}_{\tau_0,\mu\nu},$$
$$g^{(2)}_{\tau_0,\mu\nu} = \frac{16}{d^2}\widehat{T}^{\alpha}_{\tau_0,\mu}\widehat{T}_{\tau_0,\alpha,\nu} + \frac{4}{d}\xi_{\tau_0}\widehat{T}_{\tau_0,\mu\nu} + \frac{4}{d}\chi_{\tau_0}g_{\mu\nu}$$

There are some exceptional cases in which the whole expansion truncates at low orders. When $d=4$, $a=1/2$ and $b=1$, if the matrix $\mathbf{T}_{\tau_0}$ admits two independent eigenvalues $\{\ell_0, \ell_1\}$, each of multiplicity 2,

$$g_{\tau,\mu\nu} = h_{\mu\nu} = g_{\mu\nu} + (\tau - \tau_0)\left[T_{\tau_0,\mu\nu} - \frac{1}{2}\mathrm{tr}\left[\mathbf{T}_{\tau_0}\right] g_{\mu\nu}\right]$$

The degeneracy condition on the stress-energy tensor turns out to be satisfied in many physically relevant examples, such as Abelian gauge theories.

$$\begin{aligned}\frac{\partial S_{U(1),\tau}}{\partial \tau} &= \int d^4\mathbf{x}\sqrt{-g}\,\mathcal{O}_{4,\tau}^{[1/2,1]} \\ &= \int d^4\mathbf{x}\sqrt{-g}\,\sqrt{\det\left[\mathbf{T}_\tau\right]}\end{aligned}$$

[Conti-Iannella-Negro-RT, Ferko-Smith-Tartaglino Mazzucchelli]

When $d = 4,\ b = 0$, the deforming operator triggers the pure-trace deformation

$$\boxed{g_{\tau,\mu\nu} = h_{\mu\nu} = g_{\mu\nu} - a\,(\tau - \tau_0)\,\mathrm{tr}\,[\mathbf{T}]\,g_{\mu\nu}}$$

and the corresponding flow equation is:

$$\frac{\partial S_{\mathrm{M},\tau}}{\partial \tau} = \frac{a}{4}\int d^4\mathbf{x}\sqrt{-g}\,\mathrm{tr}\,[\mathbf{T}_\tau]^2$$

Under the change of metric, we have, for this family $T\bar{T}$ -like operators :

$$\boxed{\left\{S_{\mathrm{M}}[g_{\mu\nu},\Phi_I] - (\tau-\tau_0)\int d^d\mathbf{x}\sqrt{-g}\,\mathcal{O}^{[a,b]}_{d,\tau_0}\right\}\Bigg|_{g=g(h)} = S_{\mathrm{M},\tau}\,[h_{\mu\nu},\Phi_I]}$$

Ricci-based gravity theories

We consider a gravitational theory described by the action in the Palatini-like framework:

$$S_{\mathrm{G},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu}\right]=\int \mathrm{d}^d\mathbf{x}\sqrt{-g}\,\mathcal{L}_{\mathrm{G},\kappa}\left(\mathbf{R}(\Gamma)\right)$$

$$\mathbf{R}=\left(g^{\mu\alpha}\mathcal{R}_{(\alpha\nu)}\right)_{\mu,\nu\in\{0,\ldots,d-1\}}$$

symmetric part, invariant under a projective transformation

$$\Gamma^{\lambda}_{\mu\nu}\to\Gamma^{\lambda}_{\mu\nu}+\xi_{\mu}\delta^{\lambda}_{\nu}$$

i.e. the metric and the connection are treated as independent dynamical fields, and the Ricci curvature tensor is considered a functional of the connection only.

In the weak-coupling limit, we require:

$$\mathcal{L}_{\mathrm{G},\kappa} = \frac{1}{2}\mathrm{tr}\left[\mathbf{R}\right] - \Lambda + O(\kappa) = \mathcal{L}_{\mathrm{EH}} - \Lambda + O(\kappa)$$

The next step involves performing the minimal coupling with matter:

$$S_\kappa\left[g_{\mu\nu}, \Gamma^\lambda_{\mu\nu}, \Phi_I\right] = S_{\mathrm{G},\kappa}\left[g_{\mu\nu}, \Gamma^\lambda_{\mu\nu}\right] + S_{\mathrm{M}}\left[g_{\mu\nu}, \Phi_I\right]$$

Note that we assume that Γ does not explicitly enter the matter action S: matter fields do not directly couple to the connection.

→ A scenario which generally holds for bosonic and Abelian gauge fields.

The classical equations of motion for this theory are obtained by performing variations with respect to each independent field:

$$\frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\mu\nu}} = \frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\rho\alpha}\mathcal{R}_{(\alpha\beta)}} \frac{\partial g^{\rho\sigma}\mathcal{R}_{(\sigma\beta)}}{\partial g^{\mu\nu}} = \frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\mu\alpha}\mathcal{R}_{(\alpha\beta)}}\mathcal{R}_{(\beta\nu)}$$

Hence, requiring $\delta_g S_\kappa = 0$, one obtains

$$2\frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\mu\alpha}\mathcal{R}_{(\alpha\beta)}}\mathcal{R}_{(\beta\nu)} - \mathcal{L}_{\mathrm{G},\kappa} g_{\mu\nu} = T_{\tau_0,\mu\nu}$$

(Not the standard Einstein field equation!)

where we introduced the stress-energy tensor of the matter theory

$$T_{\tau_0,\mu\nu} := \frac{-2}{\sqrt{-g}}\frac{\delta S_{\mathrm{M}}}{\delta g^{\mu\nu}}$$

Setting $\delta_\Gamma S_\kappa = 0$ with $\mathcal{R}_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\nu\mu} - \partial_\nu \Gamma^\alpha_{\alpha\mu} + \Gamma^\alpha_{\alpha\beta}\Gamma^\beta_{\nu\mu} - \Gamma^\alpha_{\nu\beta}\Gamma^\beta_{\alpha\mu}$

one gets
$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}\left(h^{-1}\right)^{\lambda\alpha}\left(\partial_\nu h_{\mu\alpha} + \partial_\mu h_{\alpha\nu} - \partial_\alpha h_{\mu\nu}\right)$$

where we introduced the auxiliary metric tensor $h_{\mu\nu}$ defined by its inverse $\left(h^{-1}\right)^{\mu\nu}$ as

$$\frac{1}{2}\sqrt{-h}\left(h^{-1}\right)^{\mu\nu} := \sqrt{-g}g^{\mu\alpha}\frac{\partial \mathcal{L}_{\mathrm{G}}}{\partial g^{\mu\alpha}\mathcal{R}_{(\alpha\nu)}}, \qquad h := \det\left[h_{\mu\nu}\right].$$

→ the auxiliary metric h is such that the connection must be h-compatible.

RBG frame and the Einstein frame

[Olmo-Rubiera Garcia-Sanchis Alepuz (2013), Olmo-Rubiera Garcia (2022), Delsate-Steinhoff (2013), Delhom-Olmo-Orazi (2015)]

It has been demonstrated that Ricci-based gravity theories can be recast into GR ones, provided we introduce modified couplings in the matter sector of the full action.

To achieve such a result, we define the auxiliary gravitational action

$$\widehat{S}_{\mathrm{G},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu},H^{\mu}_{\nu}\right]=\int \mathrm{d}^d\mathbf{x}\sqrt{-g}\left\{\mathcal{L}_{\mathrm{G},\kappa}(\mathbf{H})+\left(g^{\mu\alpha}\mathcal{R}_{(\alpha\nu)}-H^{\mu}_{\nu}\right)\frac{\partial\mathcal{L}_{\mathrm{G},\kappa}}{\partial H^{\mu}_{\nu}}\right\}$$

Then the total action is

$$\widehat{S}_{\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu},H^{\mu}_{\nu},\Phi_I\right]=\widehat{S}_{\mathrm{G},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu},H^{\mu}_{\nu}\right]+S_{\mathrm{M}}\left[g_{\mu\nu},\Phi_I\right]$$

Performing the variation with respect to the matrix field H, we have

$$\delta_H \widehat{S}_\kappa = \int \mathrm{d}^d\mathbf{x}\sqrt{-g}\left\{\frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial H^\alpha_\beta} + \left(g^{\mu\rho}\mathcal{R}_{(\rho\nu)} - H^\mu_\nu\right)\frac{\partial^2 \mathcal{L}_{\mathrm{G}}}{\partial H^\mu_\nu \partial H^\alpha_\beta} - \frac{\partial \mathcal{L}_{\mathrm{G},\kappa}}{\partial H^\alpha_\beta}\right\}\delta H^\alpha_\beta$$

which constrains the on-shell value of $\mathbf{H}$ to

$$\mathbf{H}^\star = \mathbf{R}$$

Therefore, we have the dynamical equivalence:

$$\widehat{S}_\kappa\left[g_{\mu\nu}, \Gamma^\lambda_{\mu\nu}, (H^\star)^\mu_\nu, \Phi_I\right] \simeq S_\kappa\left[g_{\mu\nu}, \Gamma^\lambda_{\mu\nu}, \Phi_I\right]$$

The gravitational sector can be recast into the more familiar form:

$$\widehat{S}_{\mathrm{G},\kappa} = \int \mathrm{d}^d\mathbf{x}\sqrt{-g}\, g^{\mu\alpha}\mathcal{R}_{(\alpha\nu)}\frac{\partial\mathcal{L}_{\mathrm{G}}}{\partial H^\mu_\nu} + \int \mathrm{d}^d\mathbf{x}\sqrt{-g}\left(\mathcal{L}_{\mathrm{G},\kappa} - H^\mu_\nu\frac{\partial\mathcal{L}_{\mathrm{G},\kappa}}{\partial H^\mu_\nu}\right)$$

Introducing another auxiliary metric tensor h:

$$\frac{1}{2}\sqrt{-h}\left(h^{-1}\right)^{\mu\nu} = \sqrt{-g}g^{\mu\alpha}\frac{\partial\mathcal{L}_{\mathrm{G}}}{\partial H^\alpha_\nu} \quad \longleftrightarrow \quad \frac{1}{2}\sqrt{-h}\left(h^{-1}\right)^{\mu\nu} = \sqrt{-g}g^{\mu\alpha}\frac{\partial\mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\rho\alpha}\mathcal{R}_{(\rho\nu)}}$$

on-shell !

We can finally write:

$$\begin{aligned}\widehat{S}_\kappa &= \frac{1}{2}\int \mathrm{d}^d\mathbf{x}\sqrt{-h}\,\left(h^{-1}\right)^{\mu\nu}\mathcal{R}_{(\mu\nu)} + \int \mathrm{d}^d\mathbf{x}\sqrt{-g}\left(\mathcal{L}_{\mathrm{G}} - H^\mu_\nu\frac{\partial\mathcal{L}_{\mathrm{G}}}{\partial H^\mu_\nu}\right) + S_{\mathrm{M}}\left[g_{\mu\nu},\Phi_I\right]\\ &= \frac{1}{2}\int \mathrm{d}^d\mathbf{x}\sqrt{-h}\,\left(h^{-1}\right)^{\mu\nu}\mathcal{R}_{(\mu\nu)} + S_{\mathrm{M},\tau_0+\kappa}\left[g_{\mu\nu},H^\mu_\nu,\Phi_I\right]\,,\end{aligned}$$

where we defined a new matter action $S_{\mathrm{M},\tau_0+\varepsilon}$ which incorporates the residual H-dependence

Defining:

$$T_{\tau_0+\kappa,\mu\nu} := \frac{-2}{\sqrt{-h}} \frac{\delta S_{\mathrm{M},\tau_0+\kappa}}{\delta h^{\mu\nu}}$$

The equations of motion are:

$$\mathcal{R}_{(\mu\nu)} - \frac{1}{2}\left(h^{-1}\right)^{\alpha\beta}\mathcal{R}_{(\alpha\beta)}h_{\mu\nu} = T_{\tau_0+\kappa,\mu\nu}$$

and, setting $H^* = R$, the new Lagrangian for the matter is:

$$\left\{ S_{\mathrm{M}}\left[g_{\mu\nu}, \Phi_I\right] + \int \mathrm{d}^d\mathbf{x}\sqrt{-g}\left(\mathcal{L}_{\mathrm{G},\kappa} - g^{\mu\alpha}\mathcal{R}_{(\alpha\nu)}\frac{\partial\mathcal{L}_{\mathrm{G},\kappa}}{\partial g^{\mu\beta}\mathcal{R}_{(\beta\nu)}}\right)\right\}\Bigg|_{g=g(h)} = S_{\mathrm{M},\tau_0+\kappa}\left[h_{\mu\nu}, \Phi_I\right]$$

That should be compared with:

Can we find a match, with $\kappa \propto (\tau - \tau_0)$?

$$\left\{ S_{\mathrm{M}}[g_{\mu\nu}, \Phi_I] - (\tau - \tau_0)\int d^d\mathbf{x}\sqrt{-g}\,\mathcal{O}^{[a,b]}_{d,\tau_0}\right\}\Bigg|_{g=g(h)} = S_{\mathrm{M},\tau}\left[h_{\mu\nu}, \Phi_I\right]$$

In conclusion, we have the following dynamical equivalence:

$$S_{\mathrm{G},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu}\right]+S_{\mathrm{M}}\left[g_{\mu\nu},\Phi_I\right]\simeq S_{\mathrm{EH}}\left[h_{\mu\nu},\Gamma^{\lambda}_{\mu\nu}\right]+S_{\mathrm{M},\tau_0+\kappa}\left[h_{\mu\nu},\Phi_I\right]$$

Main objectives:

1) Study interesting modified gravity models → Modified matter

2) Study interesting irrelevant deformations → Modified gravity models

$T\bar{T}$ -deformed Abelian gauge theory and Eddington-inspired Born-Infeld gravity

Consider the following modified gravity action:

$$S_{\text{EiBI},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu}\right]=\frac{1}{\kappa}\int \mathrm{d}^d\mathbf{x}\left\{\sqrt{-\det\left[g_{\mu\nu}+\kappa\mathcal{R}_{(\mu\nu)}(\Gamma)\right]}-\lambda\sqrt{-g}\right\}\qquad(\lambda=1)$$

then

$$\left\{S_{\text{M}}\left[g_{\mu\nu},\Phi_I\right]+\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\left[\frac{1}{\kappa}\left(1-\frac{\sqrt{-h_\kappa}}{\sqrt{-g}}\right)-\frac{1}{2}\mathrm{tr}\left[\mathbf{T}_{\tau_0}\right]\right]\right\}\Bigg|_{g=g(h)} = S_{\text{M},\tau_0+\kappa}\left[h_{\kappa,\mu\nu},\Phi_I\right]$$

with

$$\frac{\sqrt{-h_\kappa}}{\sqrt{-g}}=\sqrt{\det\left[\mathbf{1}-\kappa\mathbf{T}_{\tau_0}\right]}$$

Using, as matter action:

$$\mathcal{L}_{U(1)}\left(g_{\mu\nu},A_\mu\right)=\mathcal{L}_{U(1)}\left(\mathrm{tr}\left[\mathbf{F}^2\right],\mathrm{tr}\left[\mathbf{F}^4\right]\right)$$

We can find h:

$$h_{\mu\nu} = g_{\mu\nu} + (\tau - \tau_0)\left[T_{\tau_0,\mu\nu} - \frac{1}{2}\mathrm{tr}\left[\mathbf{T}_{\tau_0}\right] g_{\mu\nu}\right]$$

with $(\tau - \tau_0) = \kappa$

and using the identity, valid in these degenerate cases:

$$1 - \frac{1}{2}\kappa\mathrm{tr}\left[\mathbf{T}_{\tau_0}\right] - \sqrt{\det\left[\mathbf{1} - \kappa\mathbf{T}_{\tau_0}\right]} = -\kappa^2\sqrt{\det\left[\mathbf{T}_{\tau_0}\right]}$$

we get

$$S_{\mathrm{EiBI},\tau_1} + S_{U(1),\tau_2} \simeq S_{\mathrm{EH}} + S_{U(1),\tau_1+\tau_2} \simeq S_{\mathrm{EiBI},\tau_2} + S_{U(1),\tau_1}$$

and as a special case $S_{\mathrm{EiBI},\tau} + S_{U(1),-\tau} \simeq S_{\mathrm{EH}} + S_{U(1),0}$

Starobinsky gravity (In the Palatini framework)

The modified gravity action is:

$$S_{\mathrm{Star},\kappa}\left[g_{\mu\nu},\Gamma^{\lambda}_{\mu\nu}\right]=\int \mathrm{d}^4\mathbf{x}\,\sqrt{-g}\left(\frac{1}{2}\mathrm{tr}\left[\boldsymbol{\mathcal{R}}(\Gamma)\right]+\frac{\kappa}{4}\mathrm{tr}\left[\boldsymbol{\mathcal{R}}(\Gamma)\right]^2\right)$$

The gravity equations for h, leads to:

$$\boxed{h_{\mu\nu}=g_{\mu\nu}-a\left(\tau-\tau_0\right)\mathrm{tr}\left[\mathbf{T}_{\tau_0}\right]g_{\mu\nu}} \quad \text{with} \quad a(\tau-\tau_0)=\kappa$$

Therefore the $T\bar{T}$-flow equation for the matter:

$$\frac{\partial S_{\mathrm{M},\tau}}{\partial\tau}=\frac{a}{d}\int d^d\mathbf{x}\sqrt{-g}\,\mathrm{tr}\left[\mathbf{T}_\tau\right]^2\,,\qquad S_{\mathrm{M},\tau_0}:=S_{\mathrm{M}}$$

If we use as unperturbed action:

$$\mathcal{L}\left(g_{\mu\nu},\phi,A_\mu\right)=\mathcal{D}_\mu\Phi\left(\mathcal{D}^\mu\Phi\right)^\dagger-V\left(\Phi\Phi^\dagger\right)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu},\qquad \mathcal{D}_\mu:=\partial_\mu-iA_\mu$$

we get:

$$\mathcal{L}_\tau\left(g_{\mu\nu},\phi,A_\mu\right)=\frac{\mathcal{D}_\mu\Phi\left(\mathcal{D}^\mu\Phi\right)^\dagger+a\tau\left[\mathcal{D}_\mu\Phi\left(\mathcal{D}^\mu\Phi\right)^\dagger\right]^2}{1+4a\tau V}-\frac{V}{1+4a\tau V}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

which is in full agreement with results in the cosmology-literature. In addition we also have:

$$\boxed{S_{\mathrm{Star},\tau}+S_{\mathrm{M}}\simeq S_{\mathrm{GR}}+S_{\mathrm{M},\tau}}$$

Conclusions and further results

* All the results obtained are in full agreement with the cosmology-literature *

1) We have explored the possibility to move $T\bar{T}$-like deformations, from matter to gravity and viceversa.

2) We have recovered two types of Modified Gravity Models:

a] The Eddington-inspired Born-Infeld theory (a= 2/d, b=1)

b] The Starobinsky gravity (a=1, b=0)

c] Further results: root-$T\bar{T}$ deformation of the the 2d "ghost-free massive gravity" discussed by Tolley and the Jackiw-Teitelboim related result by Dubovsky, Gorbenko, and Mirbabayi.

[Babaei-Aghbolagh, He, Morone, Ouyang, RT]

Thank you for your attention!