Chiral Basis for Qubits and Spin-Helix Decay

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Outline of the talk

Correlation functions of the XX(Z) quantum spin chain

- Equilibrium examples for XX
  - longitudinal correlations
  - high-T asymptotics of the transverse correlation functions at any fixed space-time point
- Quantum spin chains in cold atom experiments
  - quantum control, search for new forms of quantum matter...
  - stationary states with long range order
- Non-equilibrium example for XX – chiral basis and spin-helix decay
  - Baxter's work on XYZ: use of chiral basis / spin helices unavoidable
  - for XX eigenstates: S^z basis possible, but here chiral basis better
- Summary

Prime example of an integrable spin chain Hamiltonian

- The XXZ model

$$H_N(\Delta) = J \sum_{j=1}^{N} \left(\sigma_{j-1}^x \sigma_j^x + \sigma_{j-1}^y \sigma_j^y + \Delta \sigma_{j-1}^z \sigma_j^z \right) - \frac{h}{2} \sum_{j=1}^{N} \sigma_j^z$$

  $J > 0$, $h \in \mathbb{R}$, $\Delta = \cos(\gamma) \in \mathbb{R}$

- Mission: calculate

$$\langle \sigma_1^z \sigma_{r+1}^z(t) \rangle_T \, , \quad \langle \sigma_1^- \sigma_{r+1}^+(t) \rangle_T \, , \quad \dots$$

  explicitly for all values of r, t, T and h!

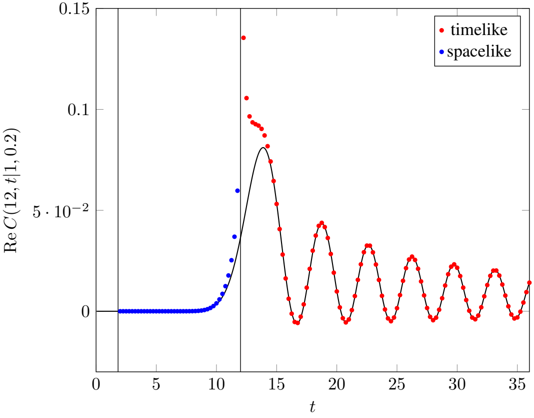
- State of the art: Finite temperature dynamical correlation functions of Yang-Baxter integrable lattice models are largely unknown. Partial exception: the XX model, $H_{XX} = H_N(0)$
- Longitudinal two-point functions of the XX model [NIEMEIJER 67, GKKKS 17]

$$\langle \sigma_1^z \sigma_{r+1}^z(t) \rangle_T - \langle \sigma_1^z \rangle^2 = \left[\int_{-\pi}^{\pi} \frac{\mathrm{d}p}{\pi} \frac{\mathrm{e}^{\mathrm{i}(rp - t\varepsilon(p))}}{1 + \mathrm{e}^{\varepsilon(p)/T}} \right] \left[\int_{-\pi}^{\pi} \frac{\mathrm{d}p}{\pi} \frac{\mathrm{e}^{-\mathrm{i}(rp - t\varepsilon(p))}}{1 + \mathrm{e}^{-\varepsilon(p)/T}} \right]$$

  where $\varepsilon(p) = h - 4J\cos(p)$

Longitudinal correlation functions of the XX model

The simple expression for the longitudinal correlations functions can be analyzed numerically and asymptotically by means of the saddle point method



Real part of the connected longitudinal two-point function of the XX chain at $r = 12$, $T = 1$, $h = 0.2$ and $J = 1/4$ as a function of time

High-T analysis –- some history

- Implication of Niemeijer's formula

$$\langle \sigma_1^z \sigma_{r+1}^z(t)\rangle_\infty = J_r^2(4Jt)$$

  where J_r, $r \in \mathbb{N}$ is a Bessel function

- A classical result by Brandt and Jacoby 1976:
  For $T \to \infty$ the transverse auto-correlation function of the XX-chain behaves as a Gaussian

$$\langle \sigma_1^- \sigma_1^+(t)\rangle_\infty = \frac{1}{2}\,\mathrm{e}^{-\mathrm{i}ht-4J^2t^2}$$

- What about $r > 0$ in the transverse case? Perk and Capel 1977 generalized this up to next-to-nearest neighbours, where modified Bessel functions occur

An explicit example beyond Perk and Capel 1977: Göhmann, Kozlowski, Suzuki 2020

$$\big\langle \sigma_1^- \sigma_4^+(t)\big\rangle_{T,h=0} \sim 16\Big(\frac{J}{T}\Big)^3 \exp\Big\{-4J^2t^2\Big\}\frac{\mathrm{I}_1(-4Jt)}{(4Jt)^5}$$

$$\times\Big\{-(4Jt)^2\,\mathrm{I}_0(-4Jt)^2 - 4Jt\,\mathrm{I}_0(-4Jt)\,\mathrm{I}_1(-4Jt) + \big((4Jt)^2+2\big)\,\mathrm{I}_1(-4Jt)^2\Big\}$$

Based on Bethe Ansatz for quantum transfer matrix, a thermal form factor series, and the analysis of a matrix Riemann-Hilbert problem

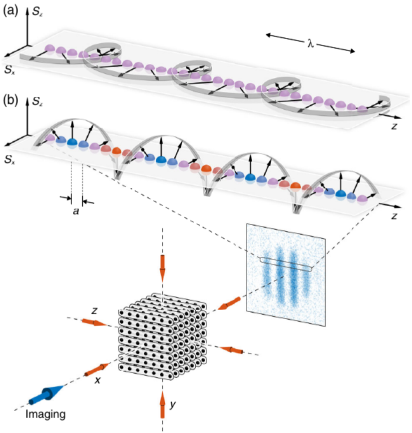


FIG. 1 from [P. N. JEPSEN ET AL. PRX **11** (2021) 041054]. Geometry of the experiment. The initial state is a transverse (a) or longitudinal (b) spin helix where the spin vector winds within the S^x-S^y plane (a) or S^z-S^x plane (b).

Deep optical lattices along the x and y directions create an array of independent spin chains. The z lattice is shallower and controls spin dynamics along each chain

$$H_N(\Delta) = J \sum_{j=1}^{N} \left(\sigma^x_{j-1}\sigma^x_j + \sigma^y_{j-1}\sigma^y_j + \Delta \sigma^z_{j-1}\sigma^z_j \right)$$

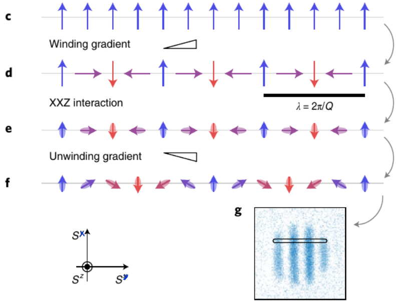


FIG. 1 from [P. N. JEPSEN ET AL. NATURE PHYS. **8** (2022) 899]. An initially spin-polarized state in the S^x direction (c) is wound into a spin helix with variable wavevector Q using a magnetic field gradient (black triangles). Here, we illustrate a winding of $Qa = \pi/2$ (d). This state evolves under the XXZ Heisenberg Hamiltonian (e). After unwinding the remaining spin modulation to a resolvable wavevector (f), the local S^x magnetization is imaged in situ (g) where dark blue indicates spins along $+S^x$. Only the S^x and S^y components of the spin are shown in c–f.

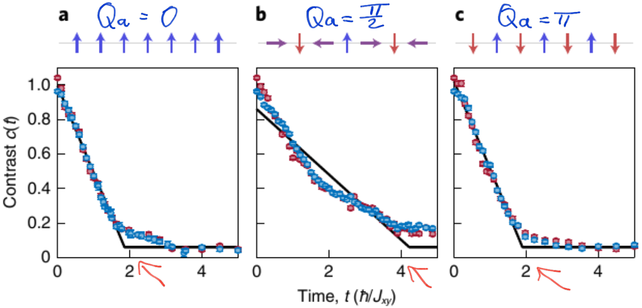


FIG. 2 from [P. N. JEPSEN ET AL. NATURE PHYS. **8** (2022) 899]. Decay of spin-helix states. a–c, The spin-helix contrast c(t) measured for $\Delta \approx 0$ at two different lattice depths $9E_R$ (red) and $11E_R$ (blue), with corresponding spin-exchange times $\hbar/J = 1.06$ and 2.91 ms, for three wavevectors: $Qa = 0$ with all spins aligned (a), $Qa = \pi/2$ with neighbouring spins perpendicular (b), which is a many-body eigenstate for $\Delta = 0$, and $Qa = \pi$ with all spins anti-aligned (c). The decay curves at different lattice depths collapse when times are normalized in units of $\hbar/J$.

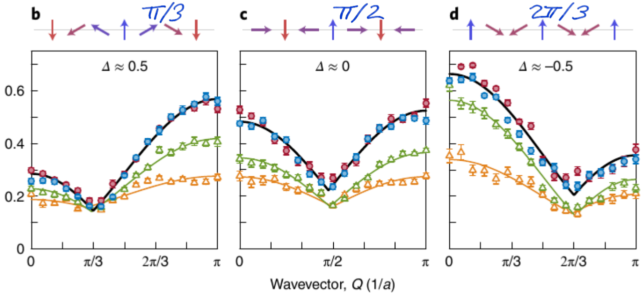


FIG. 3 from [P. N. Jepsen et al. Nature Phys. **8** (2022) 899]. Observation of phantom helix states. The decay rate γ as a function of the wavevector Q. Minimum / eigenstate condition in accordance with the phantom conditon

$$\Delta = \cos(Qa)$$

(from now on $a = J = \hbar = 1$)

Decay of a transverse spin helix

- Fully polarized state in x-direction

$$|\Omega_0\rangle = \begin{pmatrix}1\\1\end{pmatrix} \otimes \begin{pmatrix}1\\1\end{pmatrix} \otimes \ldots \otimes \begin{pmatrix}1\\1\end{pmatrix}, \quad \langle\Omega_0|\vec{\sigma}_n|\Omega_0\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \vec{\sigma}_n = \begin{pmatrix}\sigma_n^x\\ \sigma_n^y\\ \sigma_n^z\end{pmatrix},$$

- Spin rotation around z-axis with linearly increasing angles Qn with position n

$$|\Omega_Q\rangle = e^{-\frac{i}{2}\Phi}|\Omega_0\rangle, \qquad \Phi = Q\sum_{n=1}^{N} n\sigma_n^z,$$

$$= \ldots \otimes \begin{pmatrix} e^{-\frac{i}{2}Qn} \\ e^{\frac{i}{2}Qn} \end{pmatrix} \otimes \ldots$$

  yields spin helix state representing a helix in the xy-plane (which is commensurate with periodic boundary conditions, if $QN = 0 \bmod 2\pi$.

- Transformation of local vector of spin operators according to 3d representation

$$e^{\frac{i}{2}\Phi}\vec{\sigma}_n e^{-\frac{i}{2}\Phi} = D(Qn)\vec{\sigma}_n, , \quad D(\varphi) = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) & \\ \sin(\varphi) & \cos(\varphi) & \\ & & 1 \end{pmatrix}$$

$$\langle\Omega_Q|\vec{\sigma}_n|\Omega_Q\rangle = \langle\Omega_0|e^{+\frac{i}{2}\Phi}\vec{\sigma}_n e^{-\frac{i}{2}\Phi}|\Omega_0\rangle = D(Qn)\langle\Omega_0|\vec{\sigma}_n|\Omega_0\rangle = \begin{pmatrix}\cos(Qn)\\ \sin(Qn)\\ 0\end{pmatrix}$$

- We are interested in the dynamics induced by the XX Hamiltonian

$$H = \sum_{n=1}^{N} \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right) = \sum_{n=1}^{N} (\vec{\sigma}_n)^T \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \vec{\sigma}_{n+1}$$

- Start with

$$\begin{aligned} \langle \Omega_Q | \vec{\sigma}_n(t) | \Omega_Q \rangle &= \langle \Omega_0 | \, e^{\frac{i}{2}\Phi} e^{itH} e^{-\frac{i}{2}\Phi} e^{\frac{i}{2}\Phi} \vec{\sigma}_n e^{-\frac{i}{2}\Phi} e^{\frac{i}{2}\Phi} e^{-itH} e^{-\frac{i}{2}\Phi} \, | \Omega_0 \rangle \\ &= \langle \Omega_0 | \, e^{it\tilde{H}} e^{\frac{i}{2}\Phi} \vec{\sigma}_n e^{-\frac{i}{2}\Phi} e^{-it\tilde{H}} \, | \Omega_0 \rangle \qquad (1) \end{aligned}$$

where

$$\tilde{H} := e^{\frac{i}{2}\Phi} H e^{-\frac{i}{2}\Phi} \qquad (2)$$

- Now we use $e^{\frac{i}{2}\Phi} \vec{\sigma}_n e^{-\frac{i}{2}\Phi} = D(Qn)\vec{\sigma}_n$ in (2)

$$\begin{aligned} \tilde{H} &= \sum_{n=1}^{N} (D(Qn)\vec{\sigma}_n)^T \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} D(Q(n+1))\vec{\sigma}_{n+1} \\ &= \sum_{n=1}^{N} (\vec{\sigma}_n)^T \begin{pmatrix} \cos(Q) & -\sin(Q) & \\ \sin(Q) & \cos(Q) & \\ & & 0 \end{pmatrix} \vec{\sigma}_{n+1} = \cos(Q) H - \sin(Q) \mathcal{J} \end{aligned}$$

with the operator of the magnetic current $\mathcal{J} = \sum_{j=1}^{N} \left(\sigma_{j-1}^x \sigma_j^y - \sigma_{j-1}^y \sigma_j^x\right)$

Decay of a transverse spin helix

- For the XX-model H and $\mathcal{J}$ commute $[\mathcal{J}, H] = 0$.
- We have $\mathcal{J}|\Omega_0\rangle = 0$
- We carry out the spin rotation in (1)

$$
\begin{aligned}
\langle\Omega_Q|\vec{\sigma}_n(t)|\Omega_Q\rangle &= <\Omega_0|\,\mathrm{e}^{\mathrm{i}t\tilde{H}}\,D(Qn)\vec{\sigma}_n\,\mathrm{e}^{-\mathrm{i}t\tilde{H}}\,|\Omega_0\rangle \\
&= <\Omega_0|\,\mathrm{e}^{-\mathrm{i}t\sin(Q)\mathcal{J}+\mathrm{i}t\cos(Q)H}\,D(Qn)\vec{\sigma}_n\,\mathrm{e}^{-\mathrm{i}t\cos(Q)H+\mathrm{i}t\sin(Q)\mathcal{J}}\,|\Omega_0\rangle \\
&= <\Omega_0|\,\mathrm{e}^{\mathrm{i}t\cos(Q)H}\,D(Qn)\vec{\sigma}_n\,\mathrm{e}^{-\mathrm{i}t\cos(Q)H}\,|\Omega_0\rangle \\
&= D(Qn)\langle\Omega_0|\vec{\sigma}_n(\cos(Q)t)|\Omega_0\rangle \quad \text{independent of } n
\end{aligned}
$$

- Dynamics of Hamiltonian is in xy plane, initial state fully polarized in x-direction $\rightarrow$ the y- and z-component of the order parameter in $|\Omega_0\rangle$ stay 0.
  (Symmetry argument with spin π-rotation around x-axes leaves H, σ_n^x and $|\Omega_0\rangle$ invariant, but not σ_n^y, σ_n^z and would not $|\Omega_Q\rangle$.)
- With $S_N(t) = \langle\Omega_0|\sigma_1^x(t)|\Omega_0\rangle$

$$
\langle\Omega_Q|\vec{\sigma}_n(t)|\Omega_Q\rangle = S_N(\cos(Q)t)\begin{pmatrix}\cos(Qn)\\ \sin(Qn)\\ 0\end{pmatrix}
$$

The spiral is not moving but globally fading away.

Chiral multi-qubit basis

- Our starting point is the winding number operator

$$V = \frac{1}{4}\sum_{k=1}^{N/2}\left(\sigma^x_{2k-1}\sigma^y_{2k} - \sigma^y_{2k}\sigma^x_{2k+1}\right)$$

  defined for an even number of qubits N

- The factorized state

$$\Psi = \varphi_1 \otimes \zeta_1 \otimes \varphi_2 \otimes \zeta_2 \otimes \ldots \otimes \varphi_{N/2} \otimes \zeta_{N/2}$$

  is an eigenstate of V provided that

$$\langle \varphi_j| = \frac{1}{\sqrt{2}}(1, \pm 1), \quad \langle \zeta_j| = \frac{1}{\sqrt{2}}(1, \mp \mathrm{i})$$

- At each link $x|x+1$ the polarization changes by $+\pi/2$ or $-\pi/2$ in the xy-plane. Each anticlockwise or clockwise rotation by $\pi/2$ adds $+1$ or -1 to the eigenvalue of $4V$. Thus, $4V|\Psi\rangle = (N-2M)|\Psi\rangle$, where M is the number of 'clockwise' rotations = kinks. Ψ is characterized by the kink positions $1 \le x_1 < \ldots < x_M \le N$ and by the polarization $\kappa = \pm$ of the first qubit φ_1. We denote this state by $\mathrm{i}^{\sum_k x_k}|\kappa; \mathbf{x}\rangle$ where $\mathbf{x} = (x_1, \ldots, x_M)$

Chiral multi-qubit basis

- The set of V eigenstates

$$\left\{ |\kappa; \mathbf{x}\rangle \middle| \kappa = \pm; M \text{ admissible}; 1 \leq x_1 < x_2 < \cdots < x_M \leq N \right\}$$

  forms an orthonormal basis, the chiral basis. Due to periodic boundary conditions admissible Ms are even (odd) if $N/2$ is even (odd)

- The chiral basis vectors are topologically non-trivial. A single kink cannot be removed from (or added to) a periodic chain by a local operation

- σ_n^z creates a kink pair at neighbouring positions $n-1$, n in a kink-free zone. A string of operators $\sigma_n^z \sigma_{n+1}^z \ldots \sigma_{n+k}^z$ creates two kinks at distance $k+1$ in a kink-free zone

$$|+; 1, k+2\rangle = \sigma_2^z \sigma_3^z \ldots \sigma_{k+2}^z |+\rangle$$

  Here $|+\rangle$ is a perfect spin helix with maximal winding number $N/4$

$$|+\rangle = |\rightarrow\uparrow\leftarrow\downarrow\rightarrow\uparrow\leftarrow\downarrow\ldots\rangle$$

- The connection between the chiral basis and the S^z eigenbasis is non-trivial

- $[V, H] = 0 \rightarrow H$ can be diagonalized in chiral basis with fixed number of kinks.

Theorem (XX chiral eigenbasis)

- *The states*

$$|\mu_M(\mathbf{p})\rangle = \sum_{1\le x_1<\ldots<x_M\le N} \chi_{\mathbf{x}}(\mathbf{p})\left\{|1;\mathbf{x}\rangle - e^{ip_1 N}|-1;\mathbf{x}\rangle\right\}$$

$$\chi_{\mathbf{x}}(\mathbf{p}) = \frac{1}{\sqrt{2N^M}} \det_{j,k=1,\ldots,M}\left\{e^{ip_j x_k}\right\}$$

$$|\kappa;\mathbf{x}\rangle = (-i)^{\sum_{j=1}^M x_j} \bigotimes_{k=1}^{x_1}\psi_k(\kappa) \bigotimes_{k=x_1+1}^{x_2} \psi_k(\kappa+2)\cdots \bigotimes_{k=x_M+1}^{N} \psi_k(\kappa+2M)$$

$$\psi_k(\kappa) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{\frac{i\pi}{2}(k-\kappa)} \end{pmatrix}$$

where M is admissible and where the chiral quasi-momenta $\mathbf{p} = (p_1, p_2, \ldots, p_M)$, satisfy either $e^{ip_j N} = 1$ or $e^{ip_j N} = -1$ for all p_j, form an orthonormal basis of XX eigenstates, $\langle\mu_M(\mathbf{p})|\mu_{M'}(\mathbf{p}')\rangle = \delta_{\mathbf{p},\mathbf{p}'}\delta_{M,M'}$

- *The corresponding energy eigenvalues are $E_{\mathbf{p}} = \sum_{j=1}^M \varepsilon_j$, $\varepsilon_j = 4\cos(p_j)$*

Calculating S_N in the chiral eigenbasis

- An important simplification when calculating $S_N(t)$ within the chiral basis results from the fact that σ_1^x acts diagonally on the basis vectors

$$\sigma_1^x|\kappa;\mathbf{x}\rangle = \kappa|\kappa;\mathbf{x}\rangle$$

for all admissible M, leading to

$$\langle\mu_{M'}(\mathbf{q})|\sigma_1^x|\mu_M(\mathbf{p})\rangle = 0\,, \quad \text{if } M \neq M' \quad (*)$$

- Inserting $\mathrm{id} = \sum_M \sum_{\mathbf{p}} |\mu_M(\mathbf{p})\rangle\langle\mu_M(\mathbf{p})|$ two times into the definition of $S_N(t)$ and using $(*)$ we obtain

$$S_N(t) = \sum_M \sum_{\mathbf{p},\mathbf{q}} \mathrm{e}^{\mathrm{i}(E_{\mathbf{p}}-E_{\mathbf{q}})t}\langle\Omega_0|\mu_M(\mathbf{p})\rangle\langle\mu_M(\mathbf{p})|\sigma_1^x|\mu_M(\mathbf{q})\rangle\langle\mu_M(\mathbf{q})|\Omega_0\rangle$$

- For the overlaps we find that $\langle\Omega_0|\mu_M(\mathbf{p})\rangle = 0$ unless $M = N/2$ which reduces the sum over M to a single term
- The remaing overlaps and matrix elements are easily calculated in the chiral eigenbasis

Calculating S_N in the chiral eigenbasis

- From now on fix $M = N/2$ and let $B_\pm = \{p \in [-\pi,\pi) \,|\, e^{ipN} = \pm 1\}$
- Then

$$\langle \Omega_0 | \mu_M(\mathbf{p}) \rangle = \frac{1}{\sqrt{2}} \left(\frac{(-i)^{N+1}}{2N} \right)^{\frac{M}{2}} \det_M G(\mathbf{p}) , \quad G_{nm}(\mathbf{p}) = e^{i2np_m}(1 + e^{-ip_m})$$

$$\langle \mu_M(\mathbf{p}) | \sigma_1^x | \mu_M(\mathbf{q}) \rangle = M^{-M} \det_M F(\mathbf{p},\mathbf{q}) , \quad F_{nm}(\mathbf{p},\mathbf{q}) = \frac{1}{e^{i(p_m - q_n)} - 1}$$

  the latter being valid if one of the vectors **p**, **q** belongs to B_+, the other one to B_-. Otherwise the matrix element vanishes
- Note that the determinants are of Vandermonde and Cauchy type as is typical for free Fermion systems

Determinant representation for S_N

Theorem (Finite-N determinant representation)

For every even N we can represent the function S_N as

$$S_N(t) = \frac{\Phi_N(t) + \Phi_N(-t)}{2} \quad \textit{where} \quad \Phi_N(t) = \det_{m,n=1,\dots,M} \phi^{(N)}_{m,n}(t)$$

with

$$\phi^{(N)}_{m,n}(t) = \frac{1}{N^2} \sum_{p \in B_+} \sum_{q \in B_-} \frac{(1 + \mathrm{e}^{-\mathrm{i}p})(1 + \mathrm{e}^{\mathrm{i}q})\, \mathrm{e}^{\mathrm{i}[2(mp - nq) + t(\varepsilon(p) - \varepsilon(q))]}}{\mathrm{e}^{\mathrm{i}(p-q)} - 1}$$

Theorem (Infinite determinant representation in thermodynamic limit)

For all $m, n \in \mathbb{Z}$ define

$$K_{m,n}(t) = \frac{t}{m-n}\big(J_{2m}(4t)J_{2n-1}(4t) - J_{2n}(4t)J_{2m-1}(4t)\big) + \frac{t}{m-n}\big(J_{2m-1}(4t)J_{2n-2}(4t) - J_{2n-1}(4t)J_{2m-2}(4t)\big) + \frac{\mathrm{i}t}{m-n-1/2}\big(J_{2m-2}(4t)J_{2n}(4t) - J_{2n-1}(4t)J_{2m-1}(4t)\big) - \frac{\mathrm{i}t}{m-n+1/2}\big(J_{2m-1}(4t)J_{2n-1}(4t) - J_{2n-2}(4t)J_{2m}(4t)\big)$$

where the J_k are Bessel functions. For all $m, n \in \mathbb{Z}_+$ let

$$A^{\pm}_{m,n}(t) = \delta_{m,n} + K_{m,n}(t) \pm K_{m,1-n}(t)$$

Then

$$S(t) = \lim_{N\to\infty} S_N(t) = \det_{m,n=1,\dots,\infty} A^{+}_{m,n}(t) \det_{m,n=1,\dots,\infty} A^{-}_{m,n}(t)$$

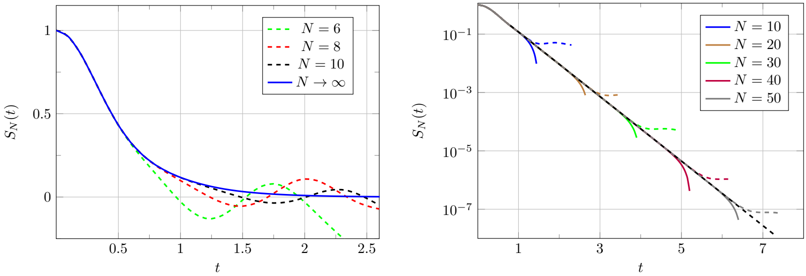
Numerical efficiency

- The above semi-infinite determinants determine $S(t)$ for all values of t
- Infinite determinants may define functions in much the same way as infinite sums or products
- The infinite determinant formula is numerically extremely efficient. E.g.

  $$S(t = 50) = 7.64483 \times 10^{-56} + 7.24454 \times 10^{-71} \times \mathrm{i}$$

  This was obtained from a truncation of the determinant to size $r = 120$. If r is further increased, the values do not change anymore. Since we know that the imaginary part is zero, the relative size of real and imaginary parts gives an estimate of the numerical accuray of the result
- Truncating the determinant at size $r = 4$ reproduces $S(t)$ with an absolute accuray of 10^{-5} for $t < 2$. The larger t the larger the size of the determinant has to be taken. $t = 2$ seems to be beyond the experimentally accessed times

Depicting the universal amplitude



Universal amplitude $S_N(t)$ of the spin-helix for different system sizes, in linear scale (left panel) and in logarithmic scale (right panel)

- Left Panel: Blue line is to be compared with Fig. 2a in [P. N. Jepsen et al. Nature Phys. **8** (2022) 899]
- Right Panel: $S_N(t)$ shows exponential decay for large times, given by the black dashed line. Coloured dashed curves show $S(t)$ with determinant size truncated to $r = [N/4]$. Curves with the same colour code correspond to the same N. Deviations from the straight line at large t are due to finite size effects

Long-time asymptotic behaviour and decay rate

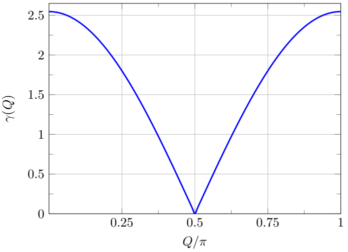
- Our numerics suggests a simple large-t asymptotics for $S(t)$

$$S(t) \sim 1.5117\, e^{-\frac{8}{\pi} t}$$

- From this asymptotics and the self-similarity we readily get the spin-helix state decay rate

$$\gamma(Q) = \frac{8}{\pi} |\cos(Q)|$$

comparable with the experimental result



Asympotic decay rate γ of the spin-helix state versus rescaled wavevector Q/π, given by $\frac{8}{\pi}|\cos(\pi x)|$. C.f. Fig. 3c in [P. N. JEPSEN ET AL. NATURE PHYS. **8** (2022) 899]

Summary

1. We have presented a view on the equilibrium and non-equilibrium dynamics of the XX chain
2. Our non-equilibrium example was the temporal decay of a transverse spin helix of wave vector Q. It was inspired by a recent cold atom experiment performed by the Ketterle group at MIT
3. The spin-helix decay is spatially uniform. The decay amplitude has a scaling form with a universal amplitude $S_N(t)$
4. We used the chiral basis to derive determinant representations for S_N and its thermodynamic limit S
5. The determinant representation of the latter is numerically highly efficient and allowed us (using also some analytic arguments that were not presented) to guess the long-time asymptotics of the function
6. Our results compare reasonably well with the experiment