Dynamics of Confining Spin Chains

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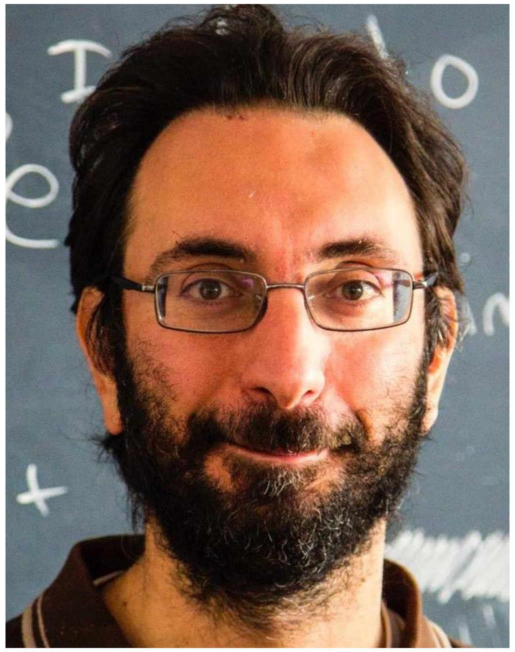
MOMENTUM OF INNOVATION

Collaborators



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Outline

1. Introduction: quantum quenches and thermalization
2. Light-cone spreading of correlation and entanglement
3. Confinement and suppression of light-cone dynamics
4. Decay of the false vacuum and Bloch oscillations
5. Local quenches and escaping fronts
6. Summary and outlook

Breaking integrability

Quantum quench: a paradigmatic non-equilibrium protocol

Initial state: ground state of some local Hamiltonian

$$H_0|\Psi(0)\rangle = \mathcal{E}_0|\Psi(0)\rangle$$

Quantum quench: a sudden change in the Hamiltonian

$$H_0 \xrightarrow[t=0]{} H \quad : \quad |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

Global quantum quench:
both H_0 and H are translationally invariant

$$\langle\Psi(0)|H|\Psi(0)\rangle \propto \mathrm{vol}(S)$$

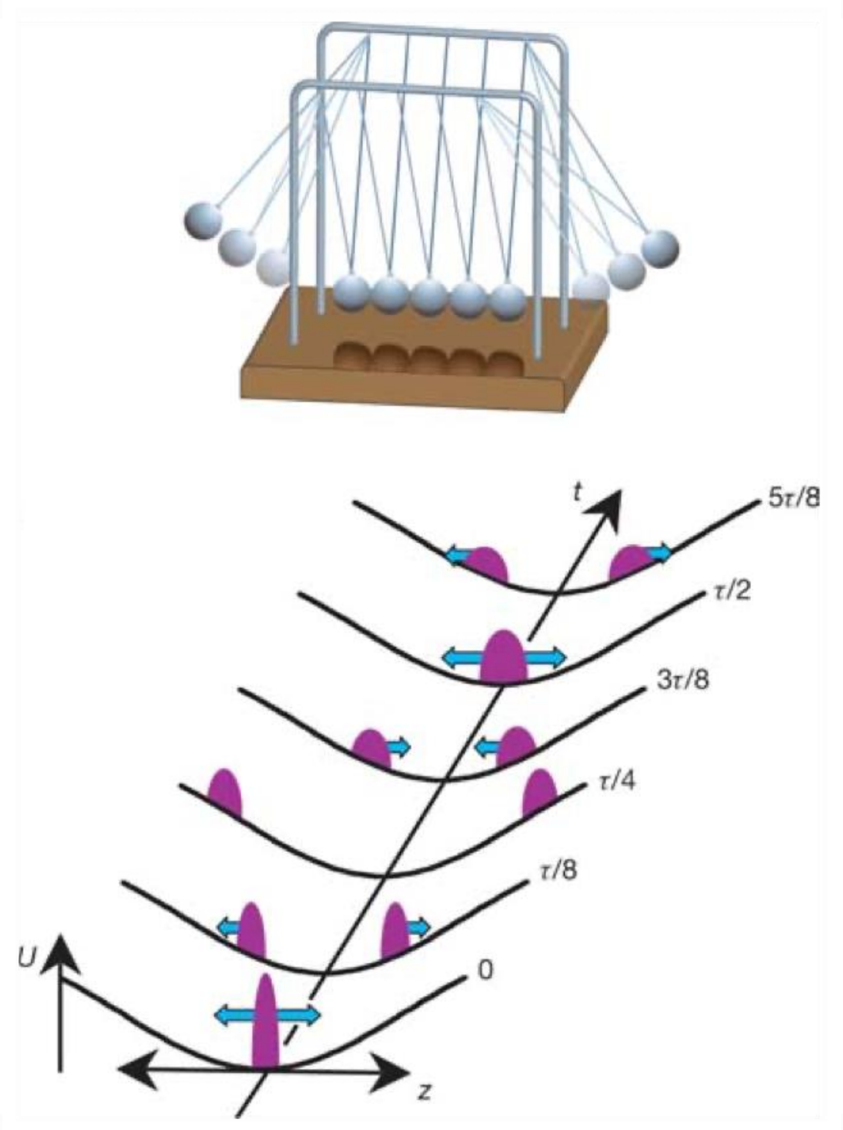
Initial state is "thermodynamical" with finite energy density

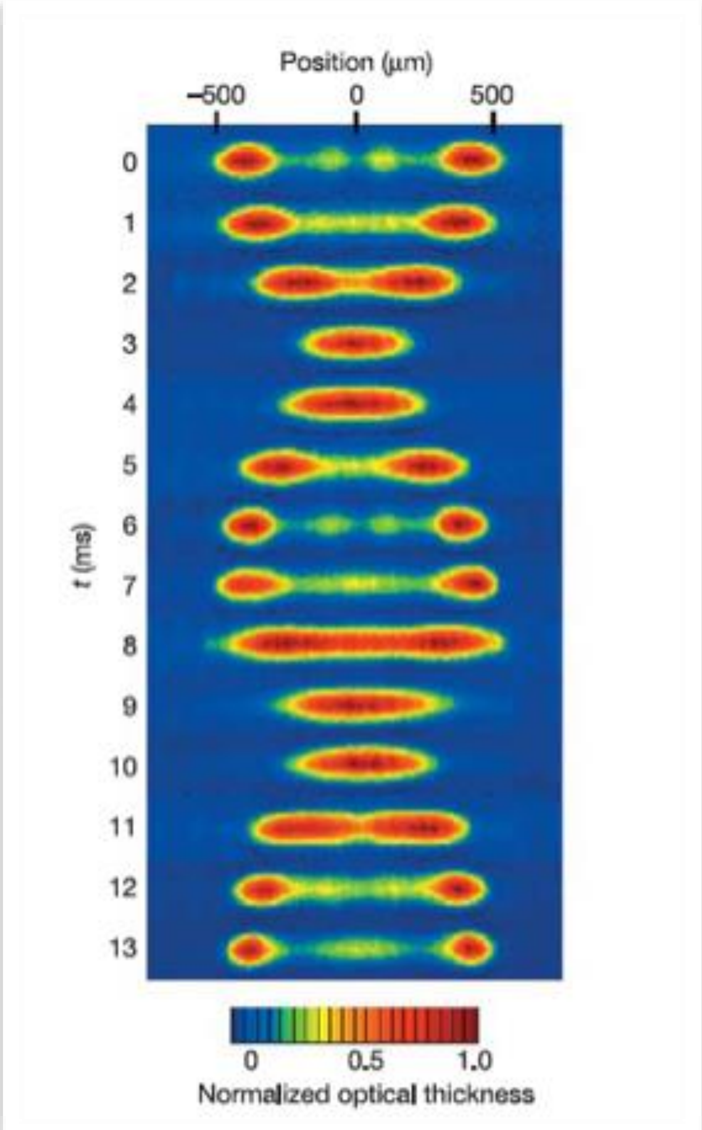
Relaxation and thermalization

Classical closed many-body systems approach equlibrium
(Boltzmann's H-theorem)

Closed quantum many-body systems:

- do they approach any sort of steady state and under what conditions?
- what is the nature of the steady state? Is it thermal?
- how does the relaxation to the steady state proceed?

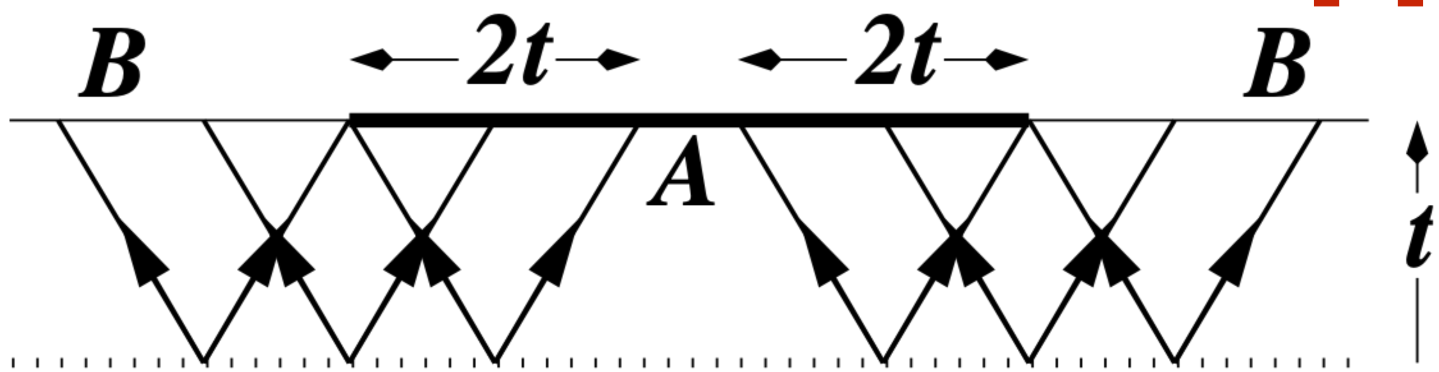




Quantum Newton's cradle experiment

T. Kinoshita et al., Nature 440, 900 (2006)

How does relaxation happen?



P. Calabrese and J. Cardy, 2005

- After quench, initial state has extensive energy
- It acts as a source of quasi-particles which propagate with momentum-dependent velocities

$$v_p = \frac{dE_p}{dp}$$

- Particles emitted from regions of the initial correlation length are correlated and entangled, while the ones emitted far from each other are incoherent
- In many systems, the velocity distribution has a maximum

$$|v_p| < v_{max}$$

(Lieb-Robinson bounds)

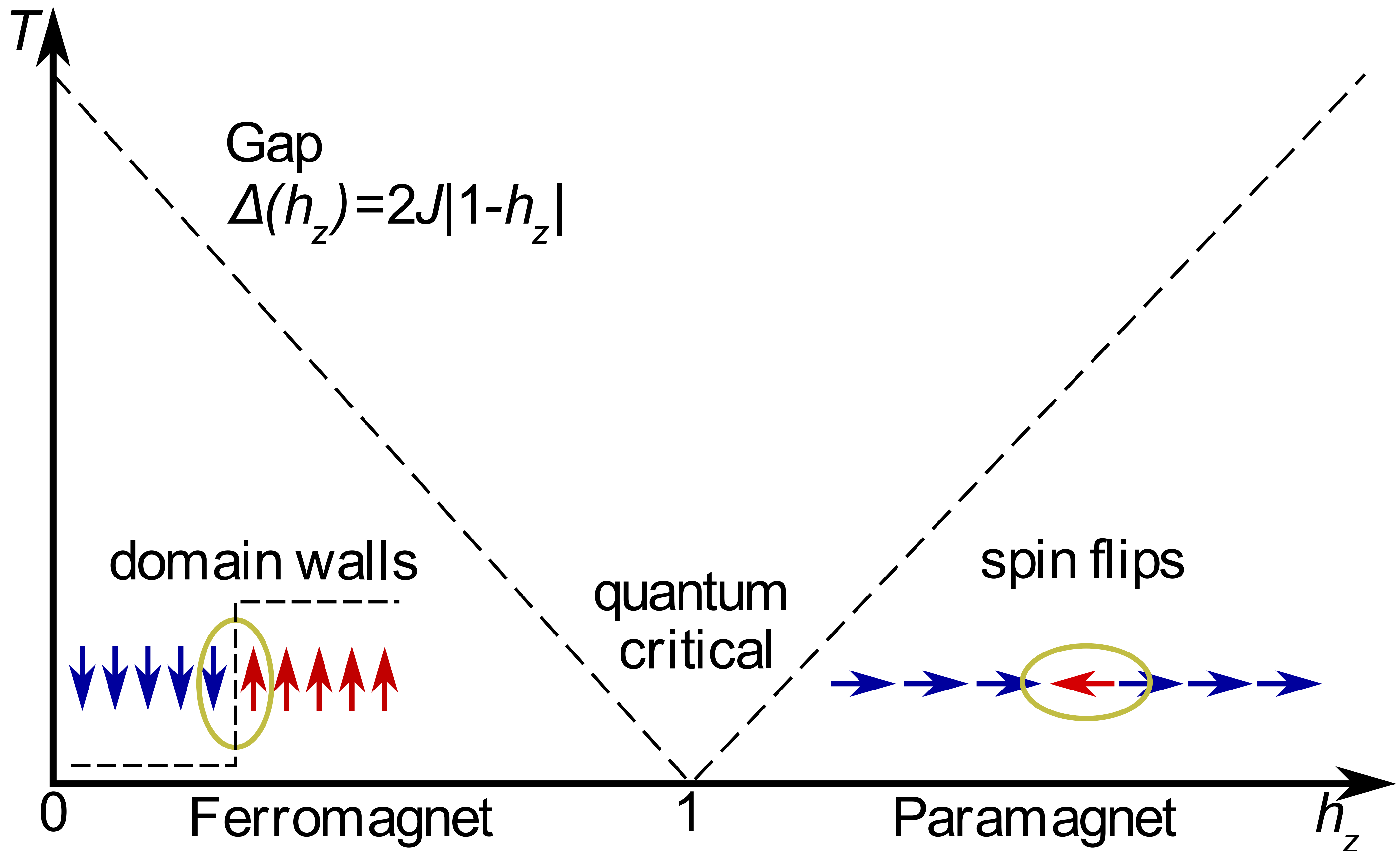
- which leads to a light-cone like spreading of correlation and entanglement.

Transverse field Ising model

$$H_{TFIM} = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z \right)$$

Model exactly solvable in terms of free fermions

$$\epsilon(k) = 2J\sqrt{1 + h_z^2 - 2h_z \cos(k)}$$



$\langle \sigma_i^x \rangle = (1 - h_z^2)^{1/8} \neq 0$ $\langle \sigma_i^x \rangle = 0$

Quantum quench in TFIM

$$H_{TFIM} = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z \right)$$

Model exactly solvable in terms of free fermions

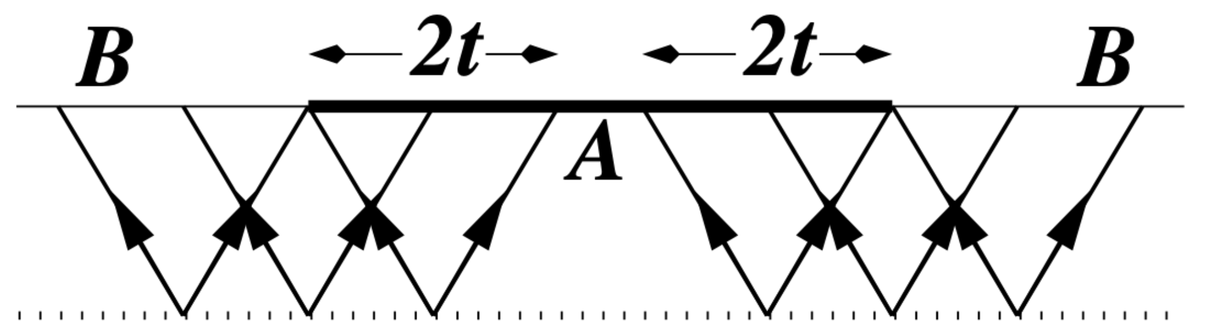
$h_z < 1$: ordered (FM) phase

$$\langle \sigma_i^x \rangle = (1 - h_z^2)^{1/8} \neq 0$$

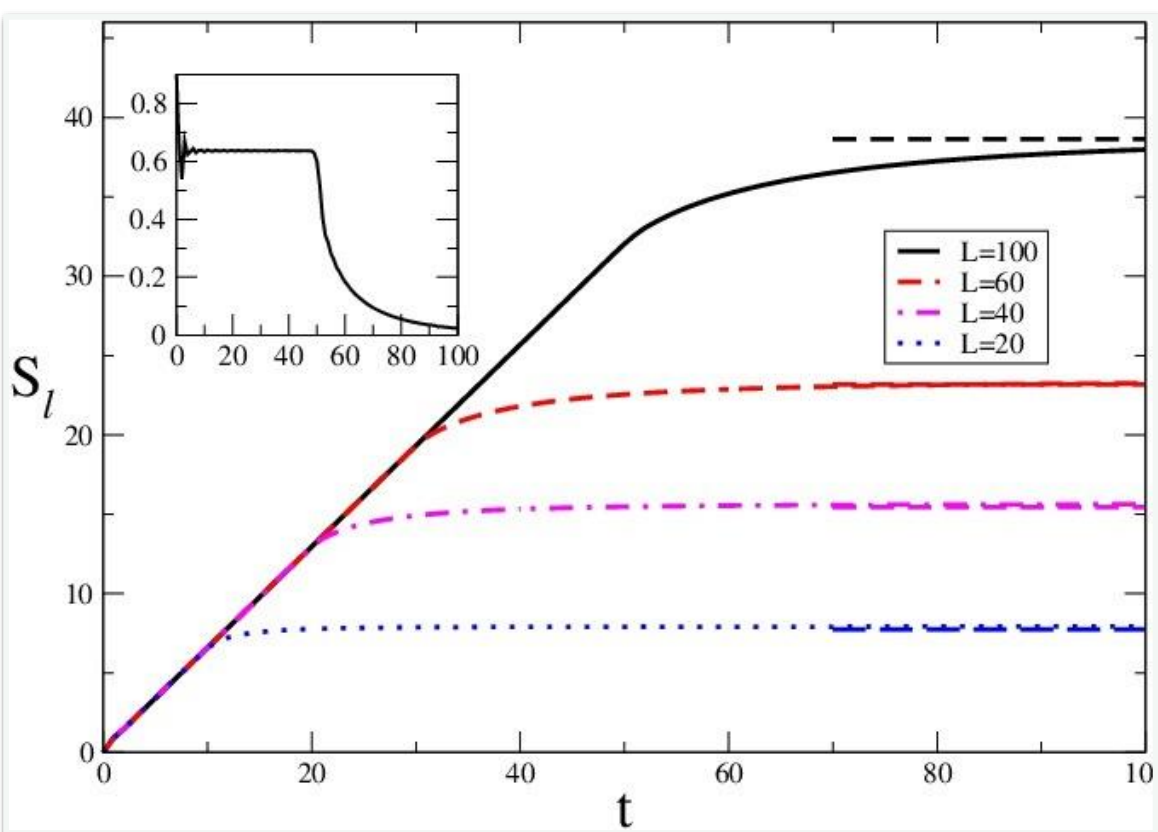
$h_z > 1$: disordered (PM) phase

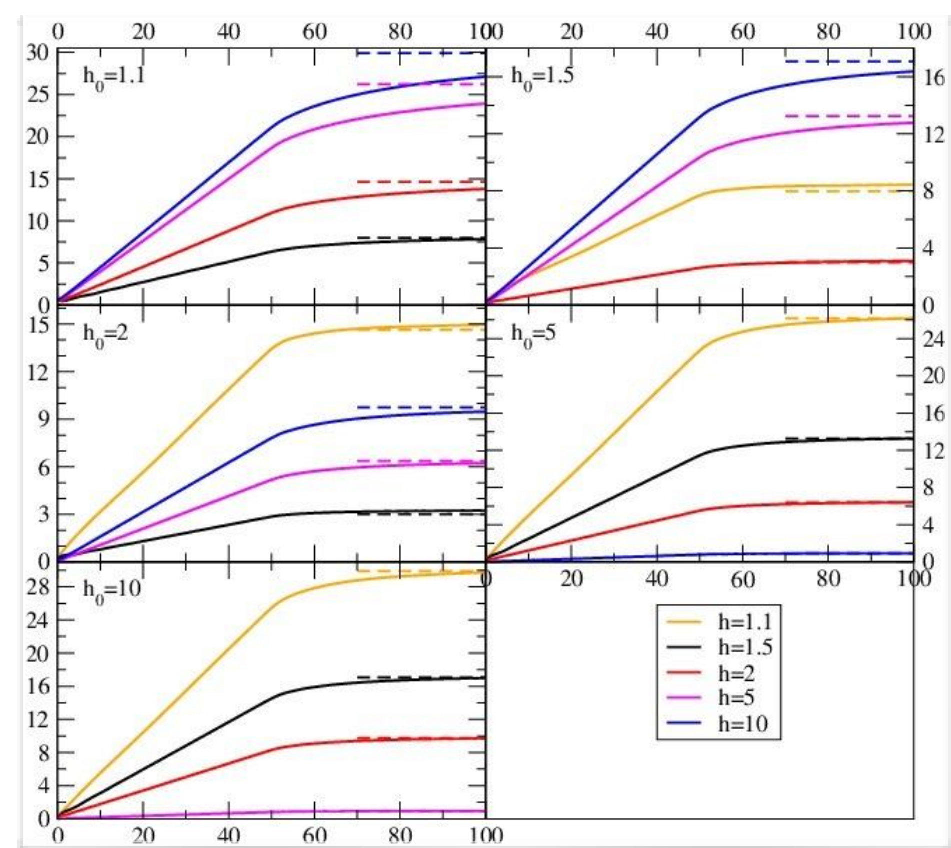
$$\langle \sigma_i^x \rangle = 0$$

Entanglement entropy between interval and rest of the system



P. Calabrese and J. Cardy, 2005





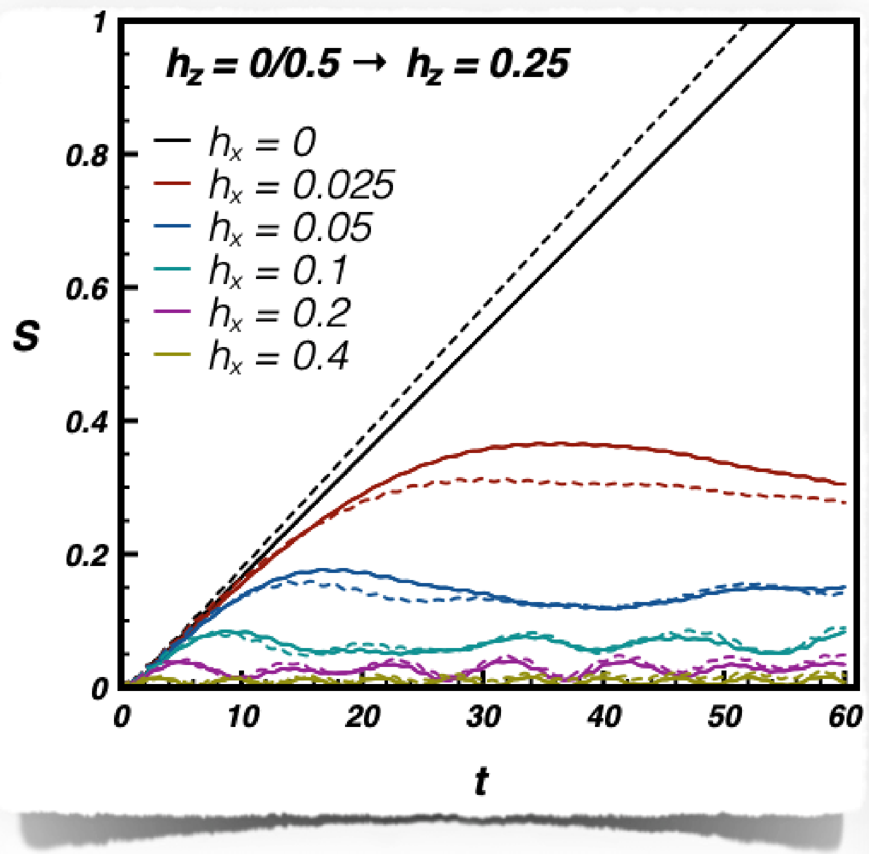
Non-integrable Ising chain in FM phase $h_z < 1$

$$H = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$

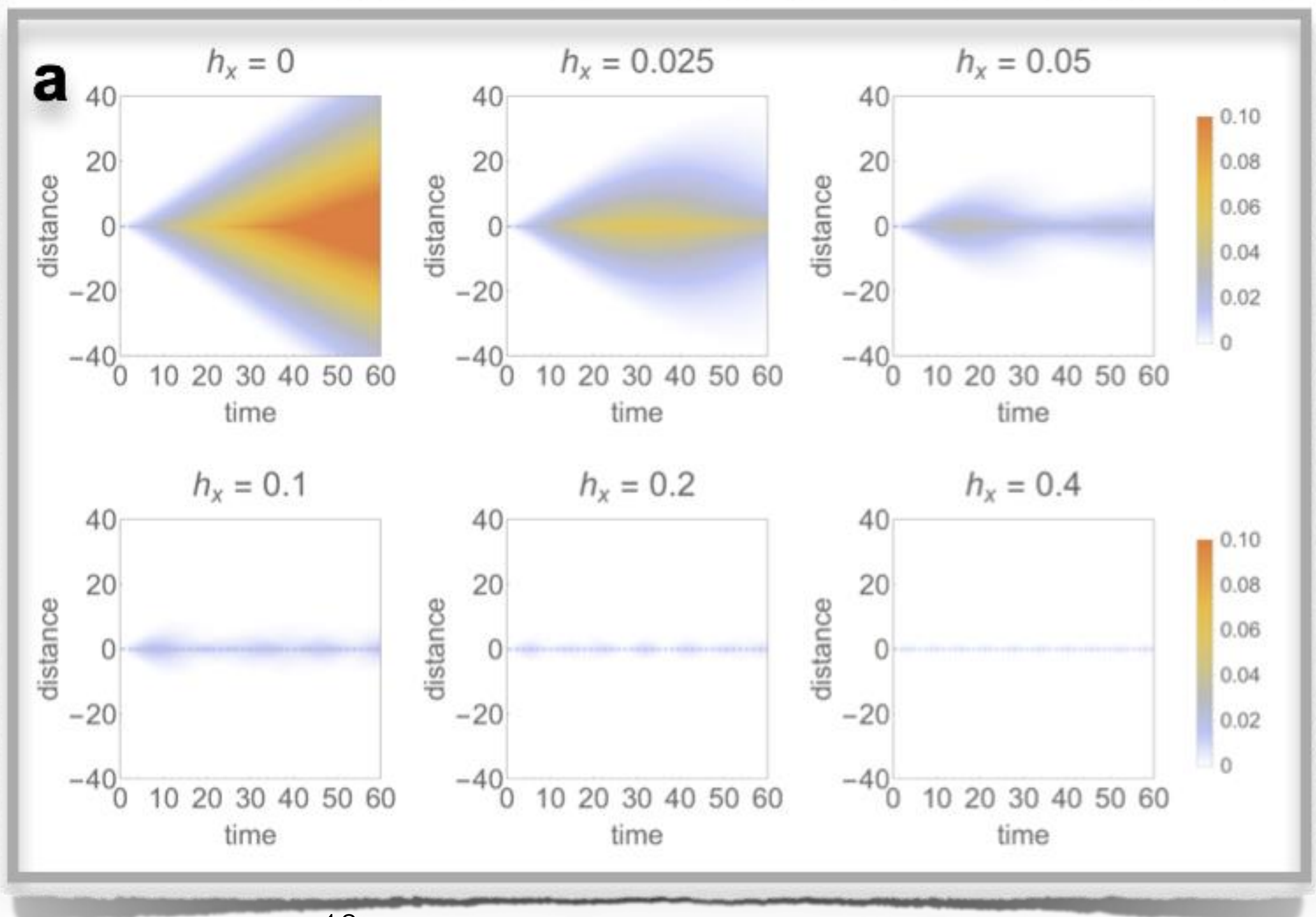
$$|\Psi(0)\rangle = \cdots \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \cdots \qquad h_x > 0$$

$\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$ **: light-cone gets suppressed!**

Entanglement entropy between left and right halves



iTEBD simulation by M. Collura (SISSA)

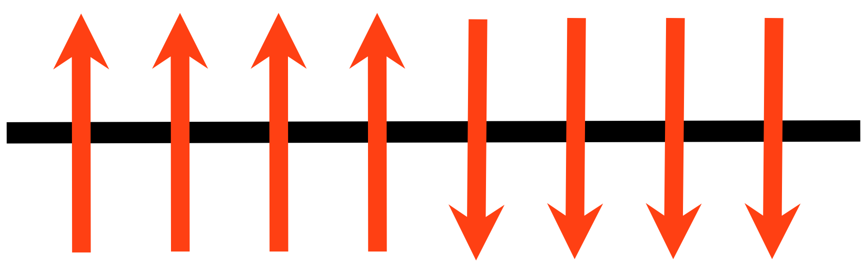


Confinement in the Ising model

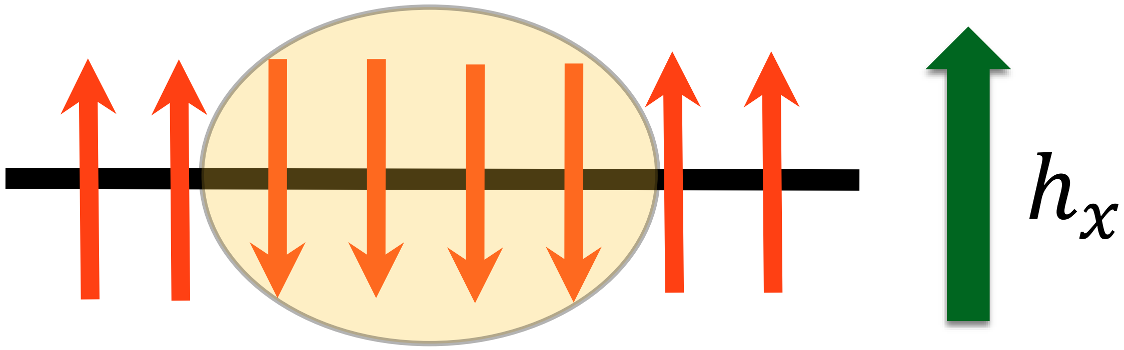
McCoy & Wu '78

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

- For $h_x = 0$ free fermions with dispersion $\epsilon(k) = 2J\sqrt{1 + h_z^2 - 2h_z \cos(k)}$
- For $h_z < 1$ (FM phase), the massive fermions correspond to domain walls separating domains of magnetisation $\sigma = \pm(1 - h_z^2)^{1/8}$



Free DW



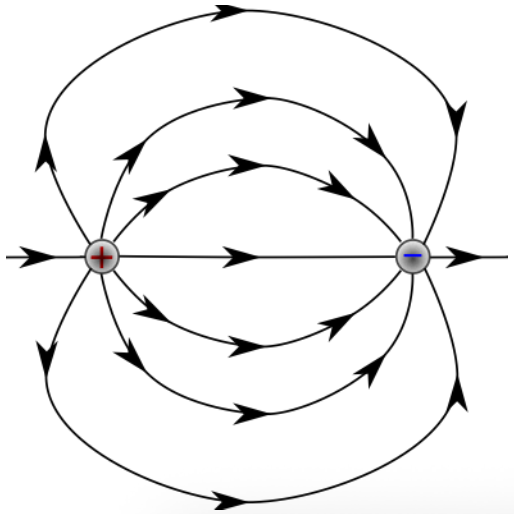
Bound state = meson

- h_x induces an attractive interaction between DWs that for small enough h_x can be approximated with a linear potential

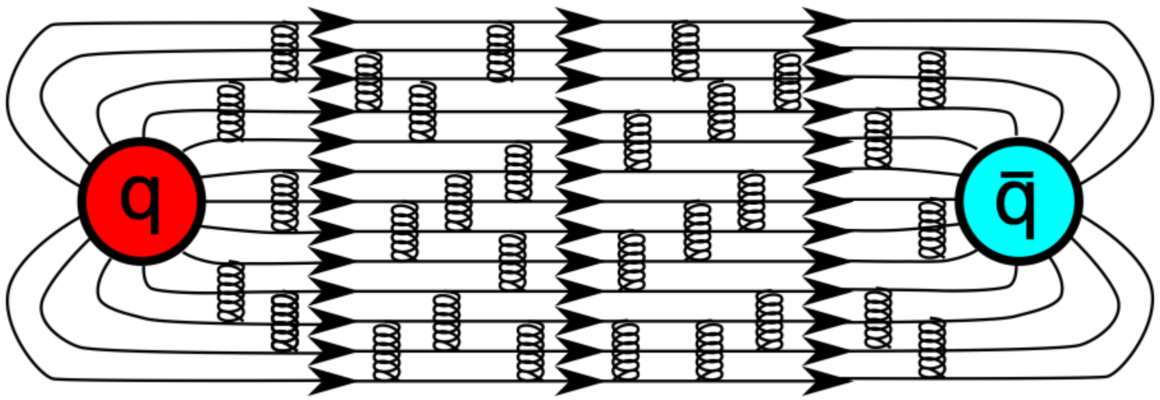
$$V(x) = 2Jh_x \sigma |x|$$

- DWs do not propagate freely but get confined into **mesons**

Quark confinement in strong interactions

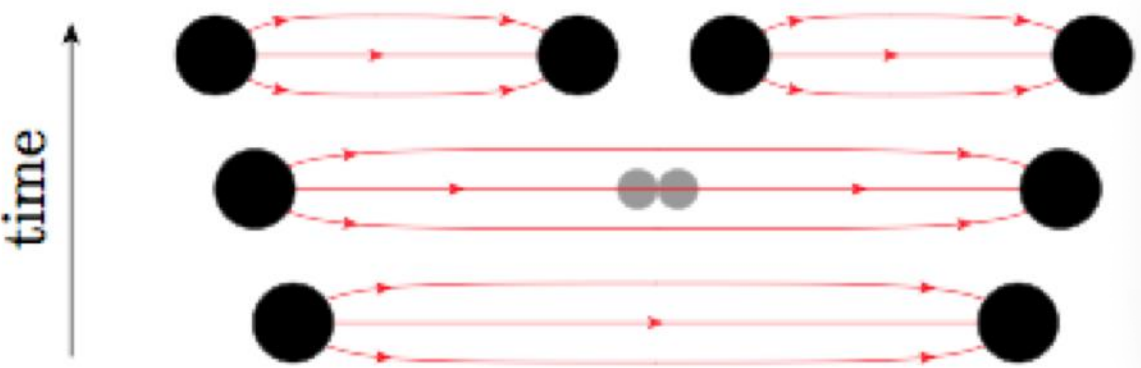


In contrast to electricity, chromoelectric field lines attract each other: strings



Flux density (= field strength) constant for large separation:

$$V(x) \propto |x|$$



Colour (quark) confinement: only colour singlet ("white") states propagate freely

these are called hadrons

Ising model: two colours — only mesons

QCD: three colours — also baryons

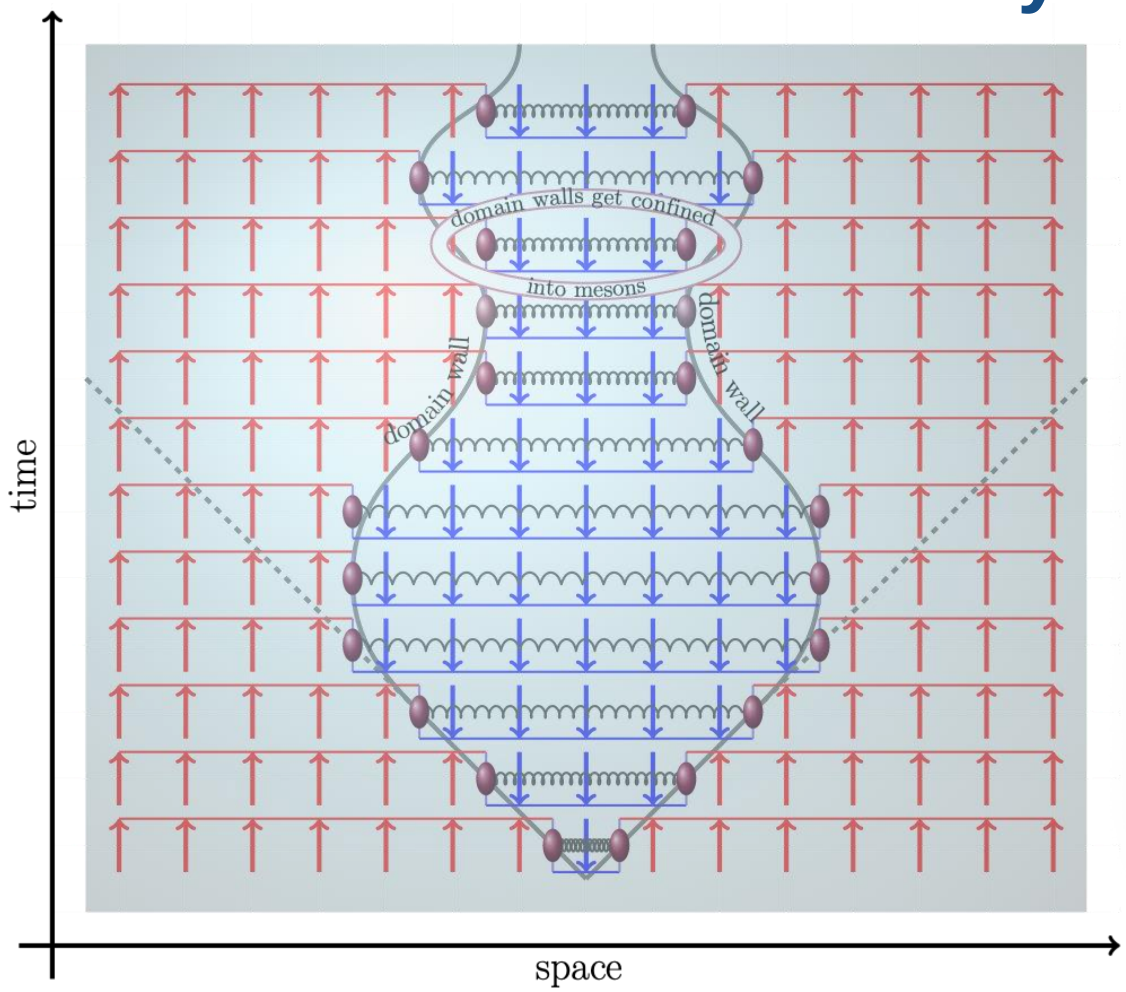
Condensed matter theory analogue with baryons: 3-state Potts

Rutkevich 2014; Lencsés & Takács 2015

Effect of confinement on time evolution

What happens if the post-quench Hamiltonian is confining?

- $|\Psi(0)\rangle$ acts as a source of quasi-particles at $t = 0$
- pairs of particles move in opposite directions with velocity v_p
- when moving away, the particles feel the attractive interaction
- the interaction eventually turns the particles back



M. Kormos, M. Collura, G. Takács, and P. Calabrese, Nature Physics 13, 246–249 (2017)

But: how can we make sure that this is not merely a just-so story?

We need real signatures linking the dynamics quantitatively to confinement!

The meson spectrum

Consider two fermions in 1D with Hamiltonian

$$H = \epsilon(k_1) + \epsilon(k_2) + \chi|x_2 - x_1| = \omega(k;K) + \chi|x|$$

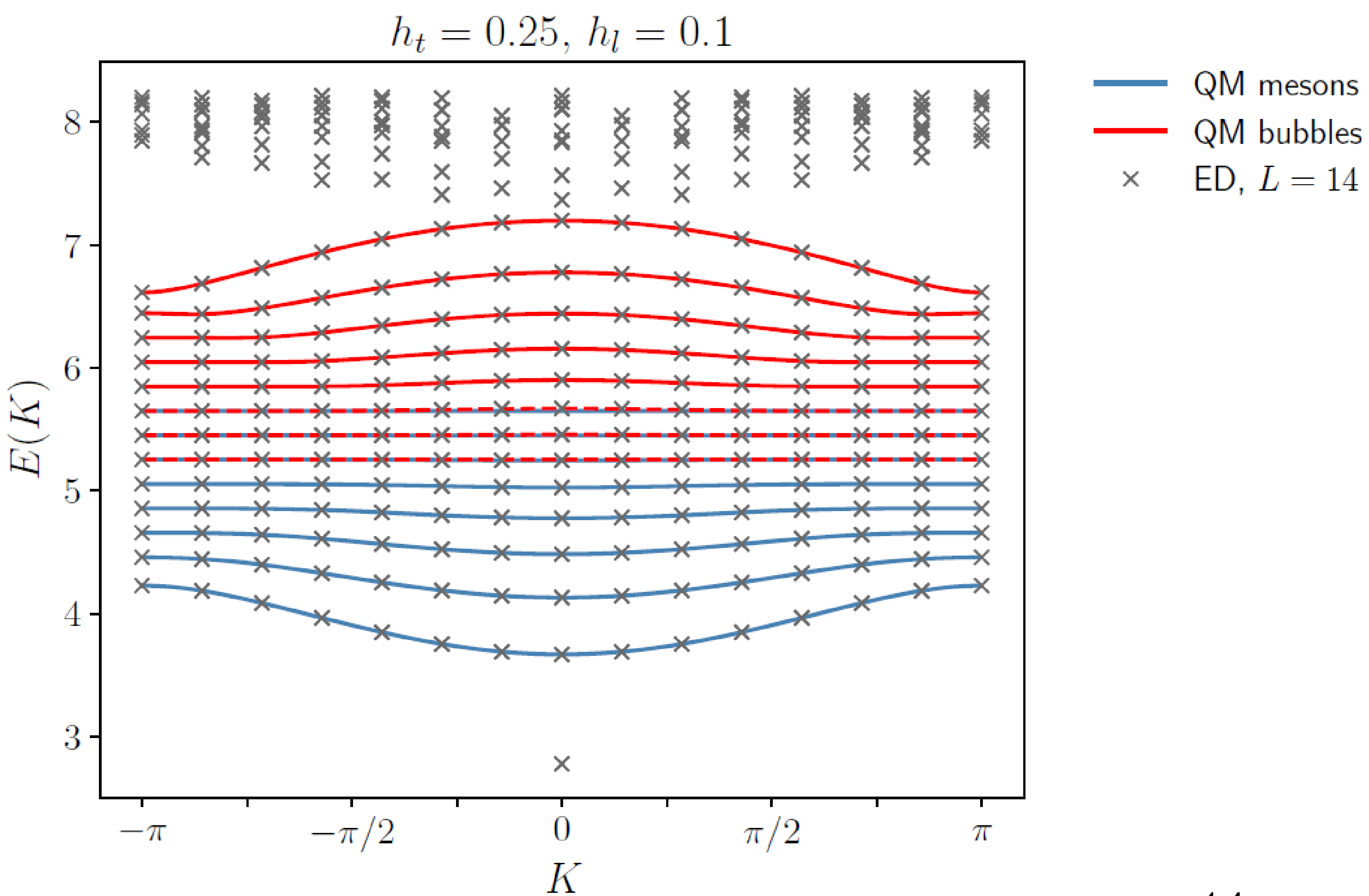
Rutkevich, 2008

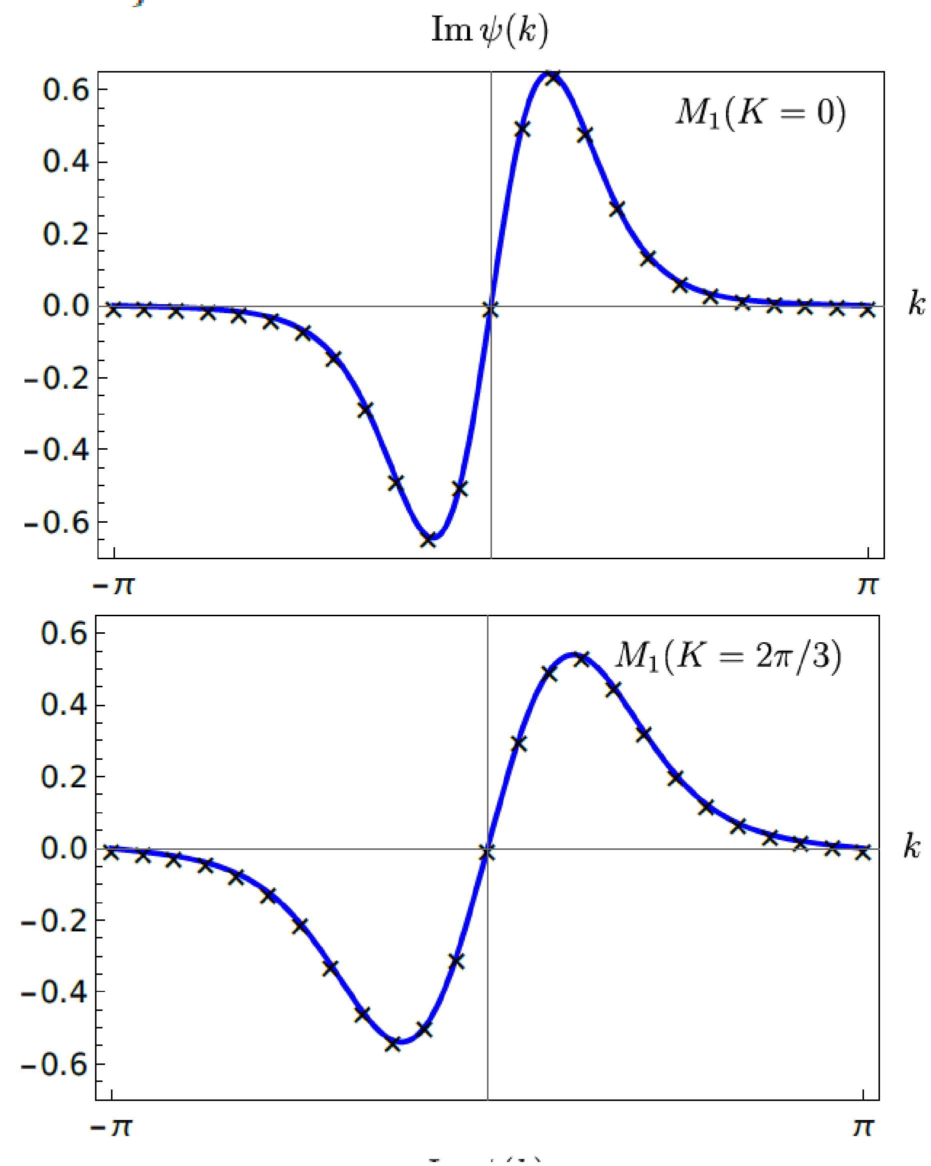
$$k_{1,2} = K/2 \pm k \qquad \chi = 2Jh_l(1-h_t^2)^{1/8}$$

Schrödinger equation → mesons labelled by species number

$$H\psi_{n,K}(x) = \sum_{x'} H(x,x';K)\psi_{n,K}(x') = E_n(K)\psi_{n,K}(x)$$

Krasznai & Takács, 2024

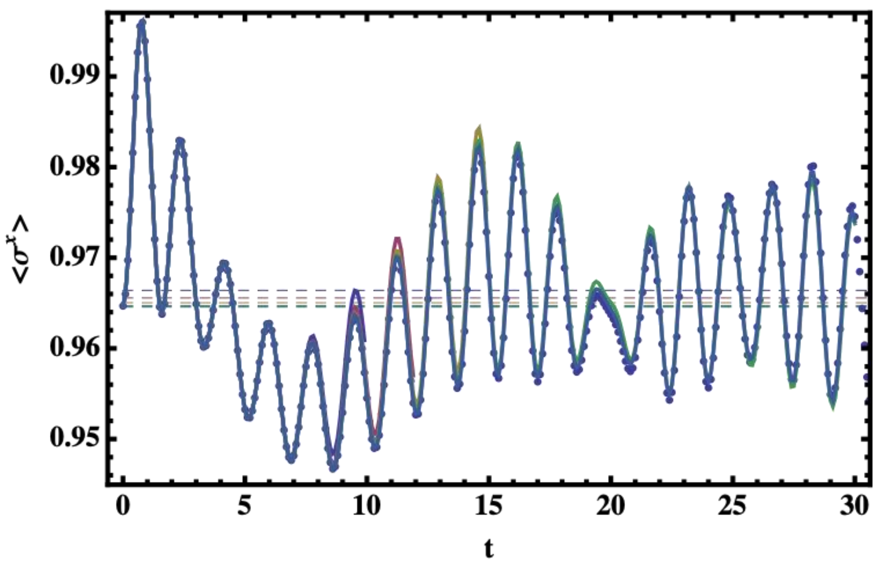




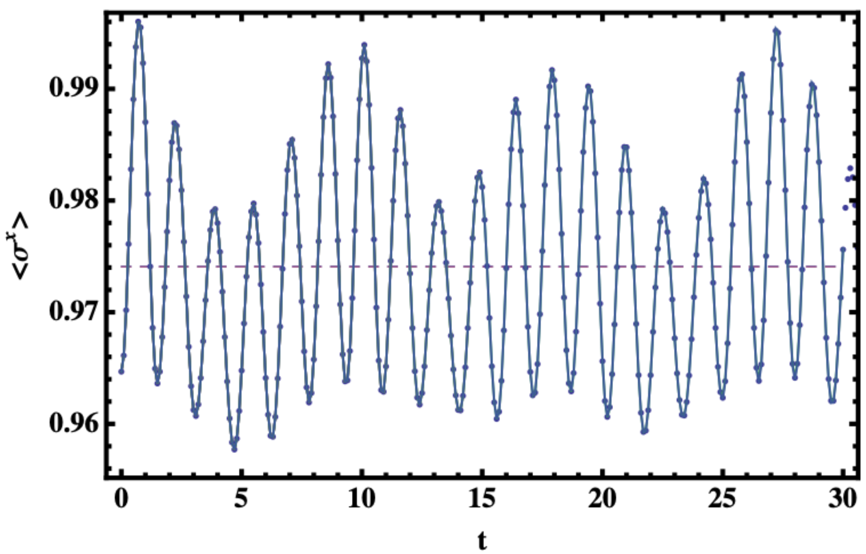
Quench spectroscopy from time evolution of magnetisation

Quenches from FM to FM: no relaxation observed!

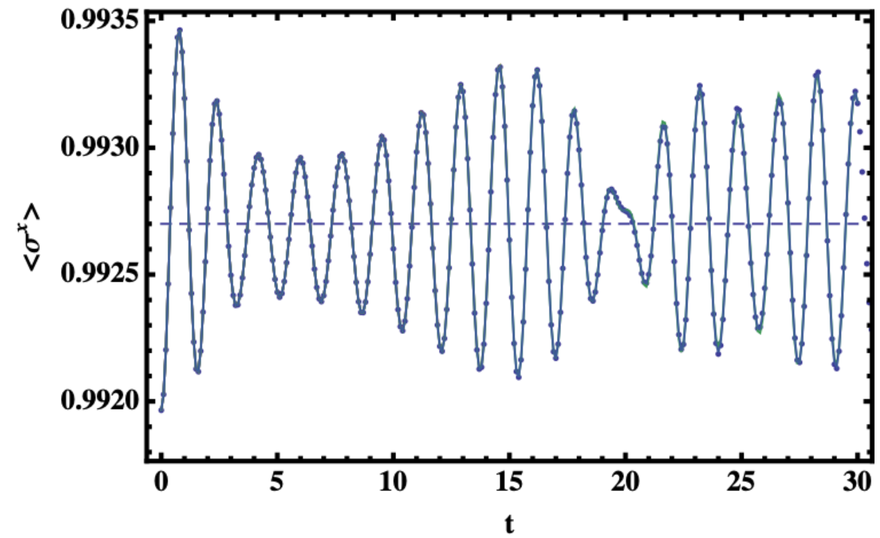
$h_z^0=0.5,\ h_x^0=0,\ h_z=0.25,\ h_x=0.1$



$h_z^0=0.5,\ h_x^0=0,\ h_z=0.25,\ h_x=0.2$

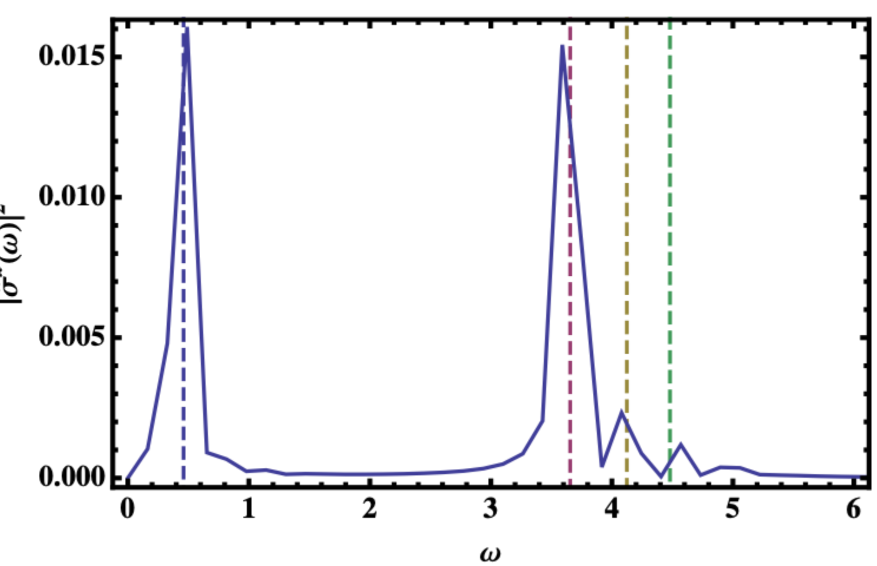


$h_z^0=0.25,\ h_x^0=0,\ h_z=0.25,\ h_x=0.1$



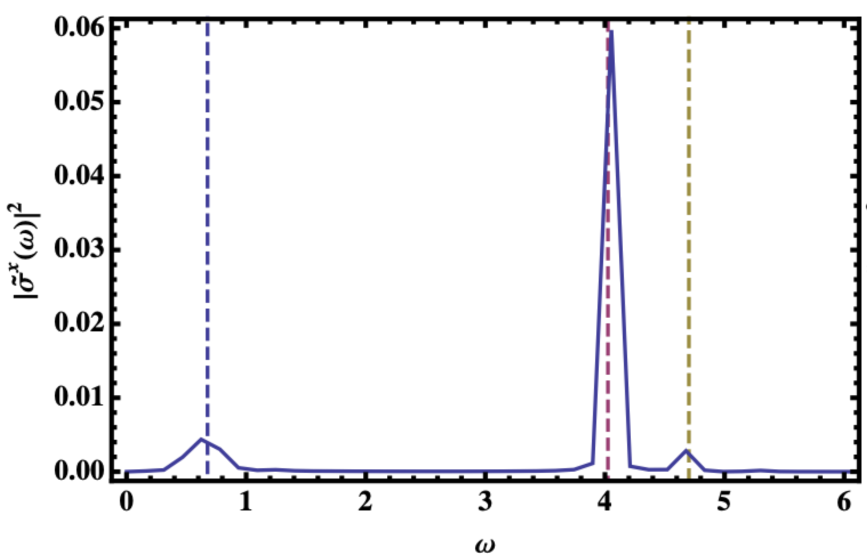
iTEBD vs. ED with L=8,10,12

Power spectrum of $\langle \sigma_x \rangle$ compared to semiclassical meson spectra



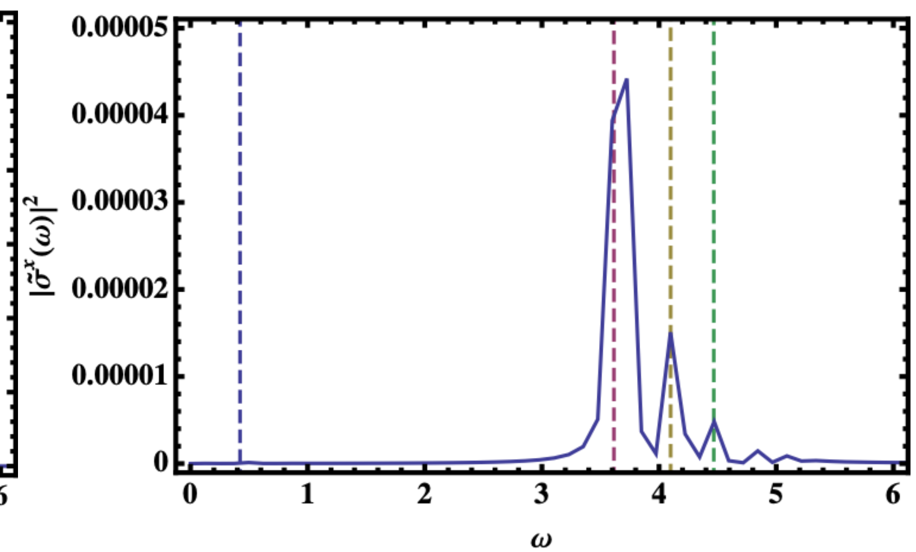
$m_2-m_1=0.46,\ m_1=3.7,$

$m_2=4.1,\ m_3=4.5$



$m_2-m_1=0.68,\ m_1=4.0,$

$m_2=4.7$



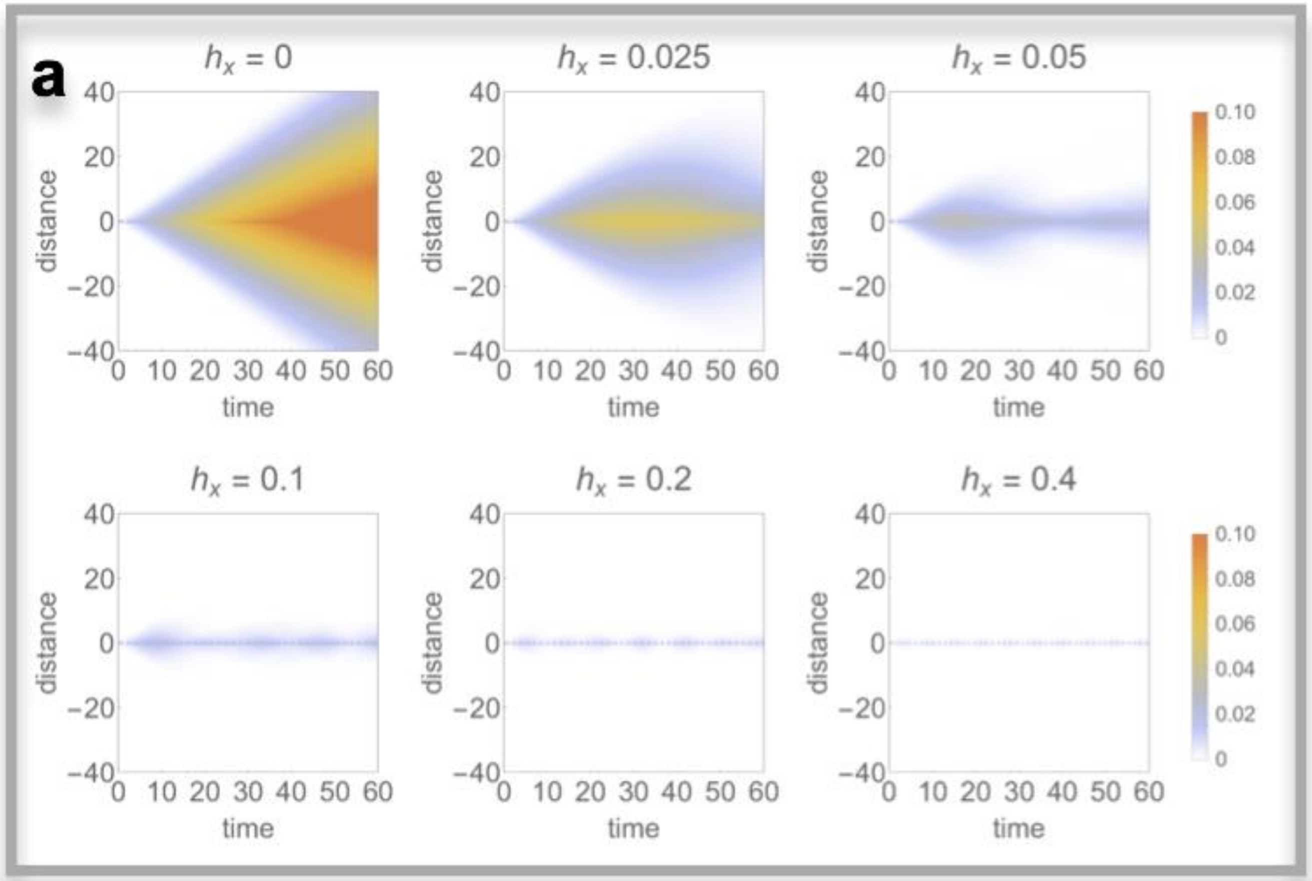
$m_2-m_1=0.46,\ m_1=3.7,$

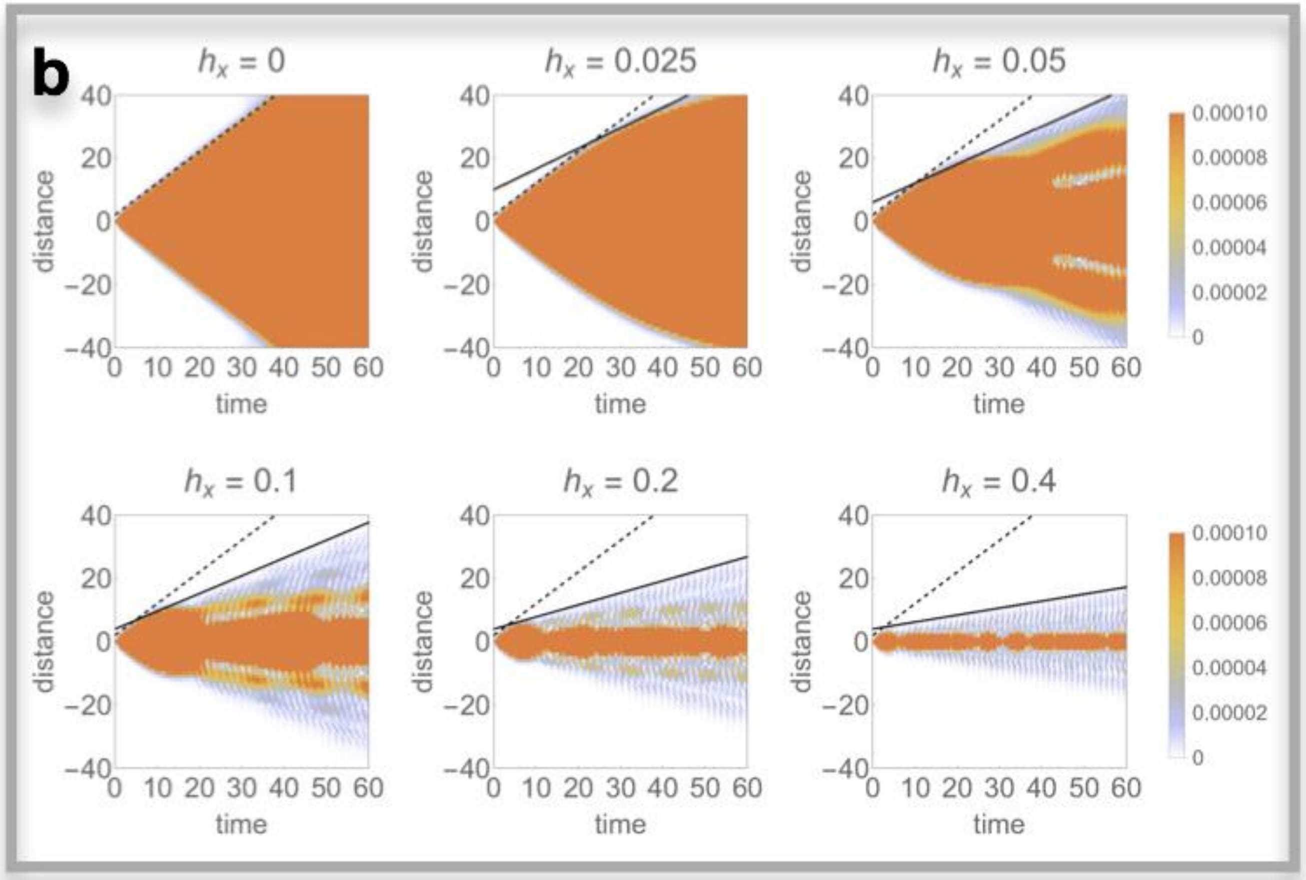
$m_2=4.1,\ m_3=4.5$

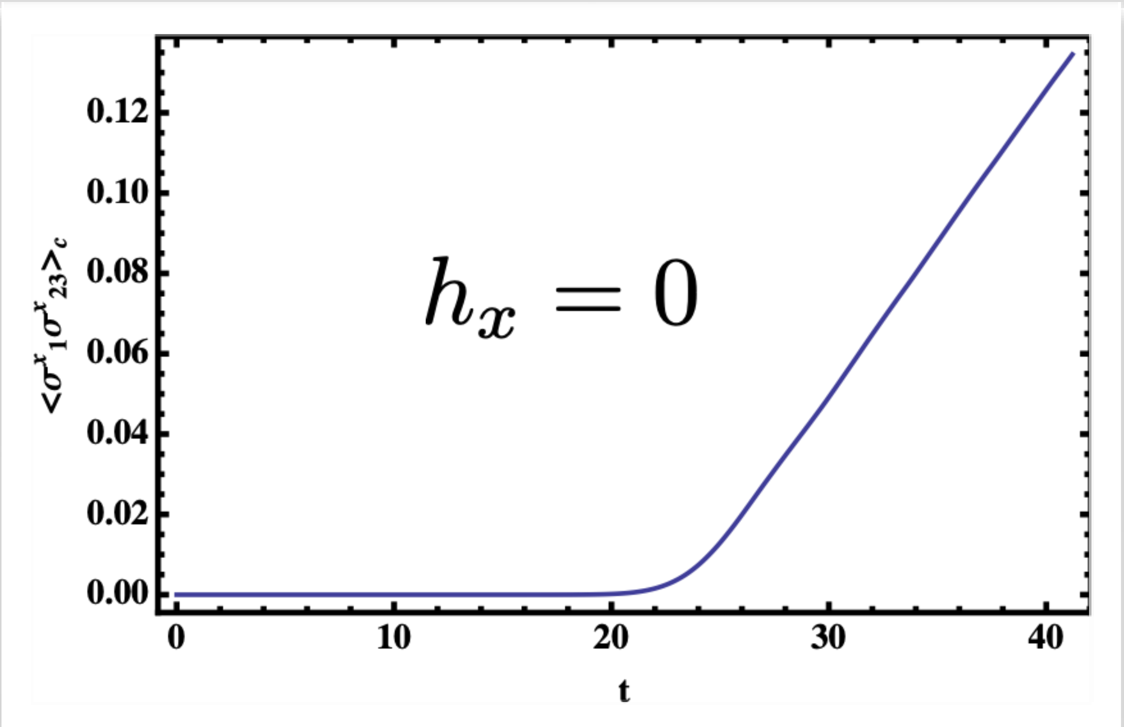
It is the mesons that are the source of the persistent oscillations!

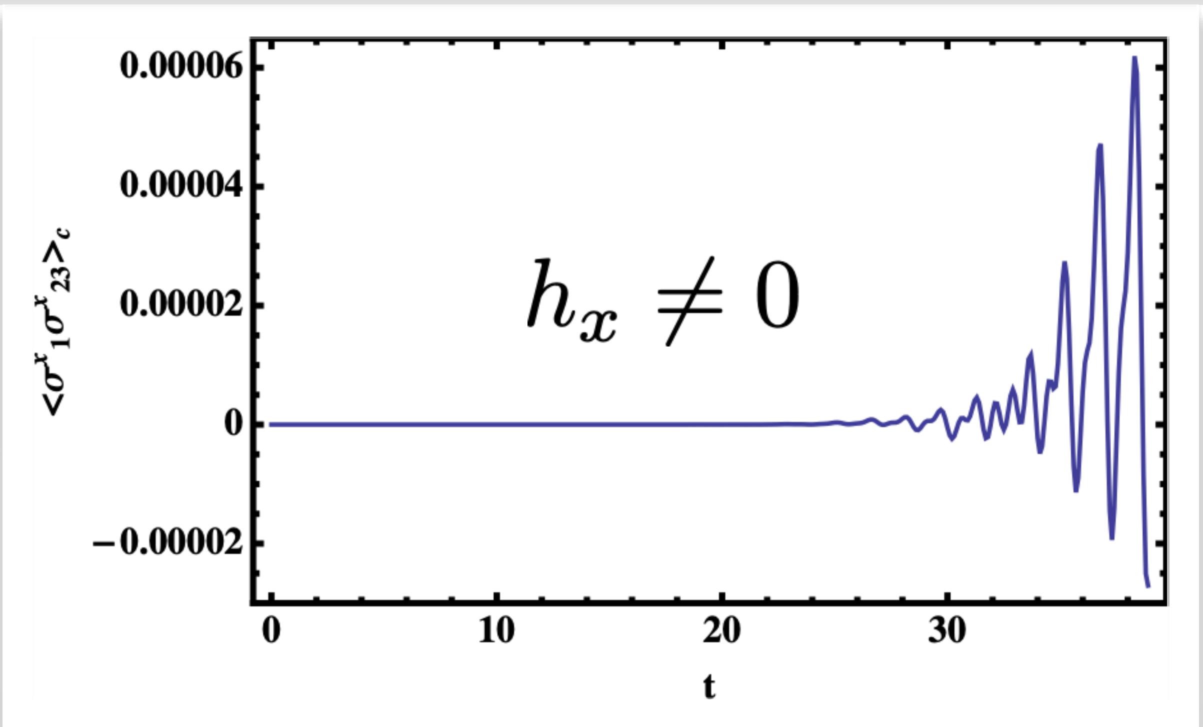
Another effect of mesons: escaping correlations

$$\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$$



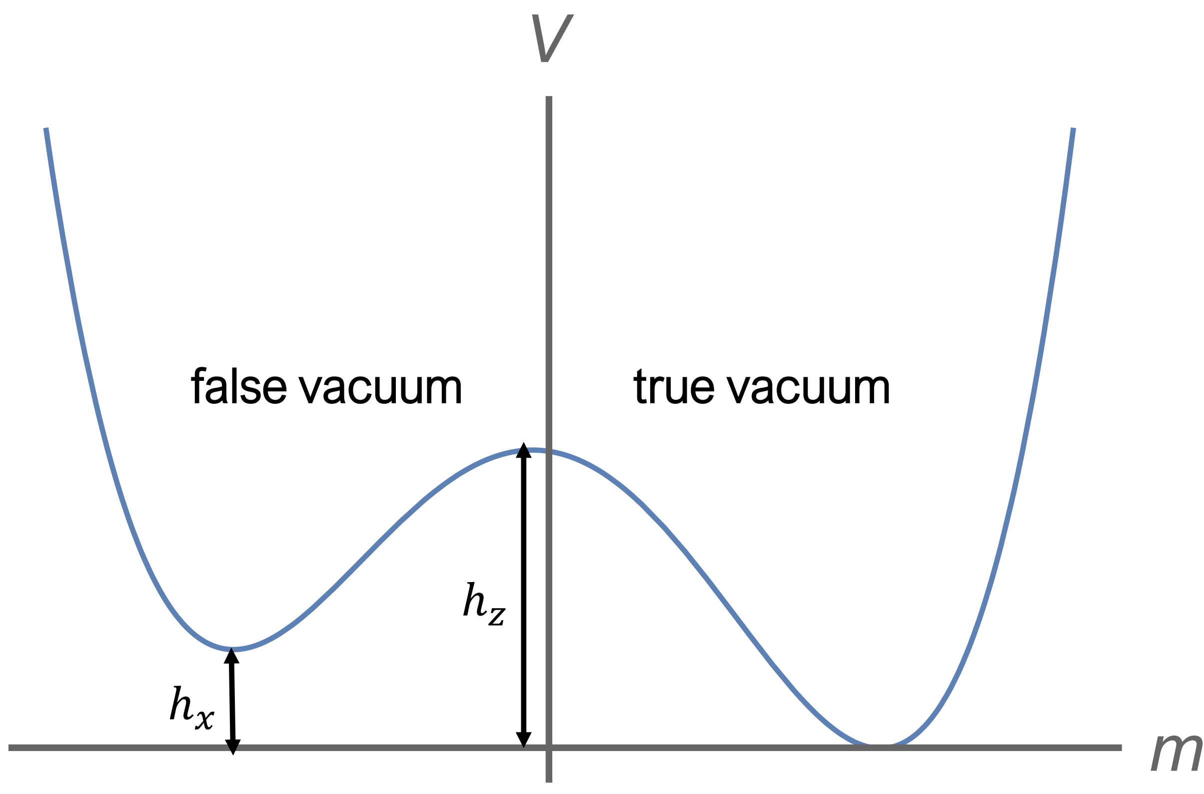






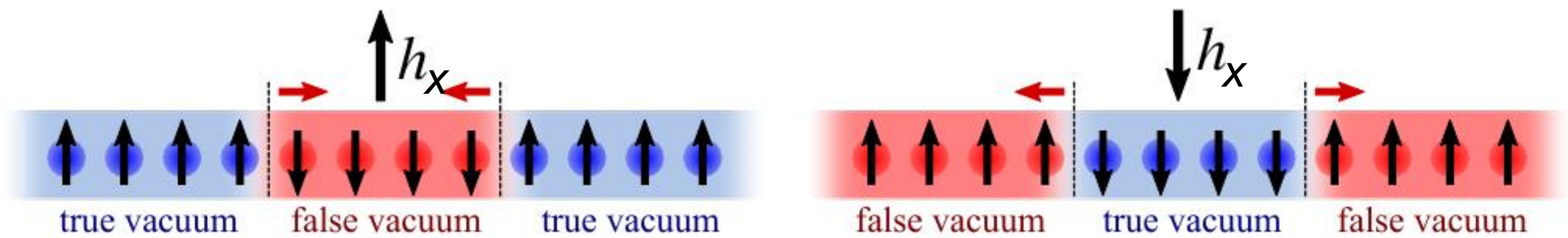
Decay of the false vacuum

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$



So far: confining quench –
h_x parallel to initial magnetisation

Other option: anti-confining quench



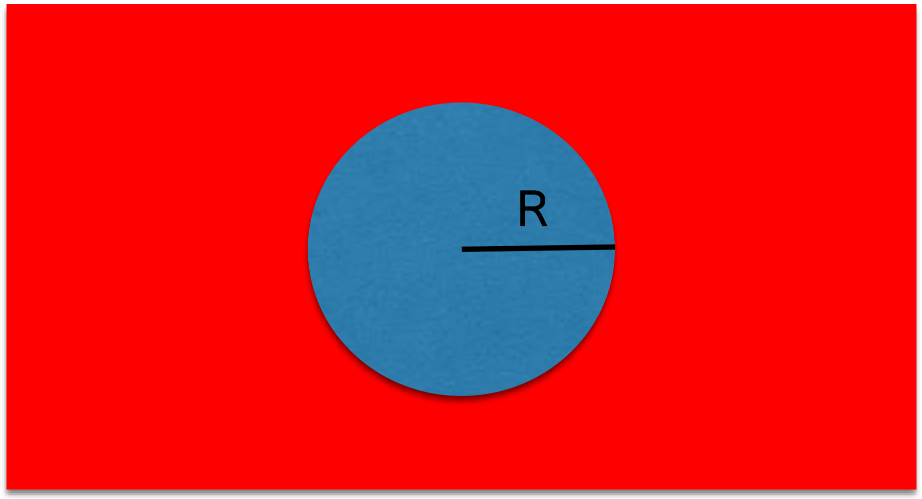
(a) Confining quench (b) Anti-confining quench

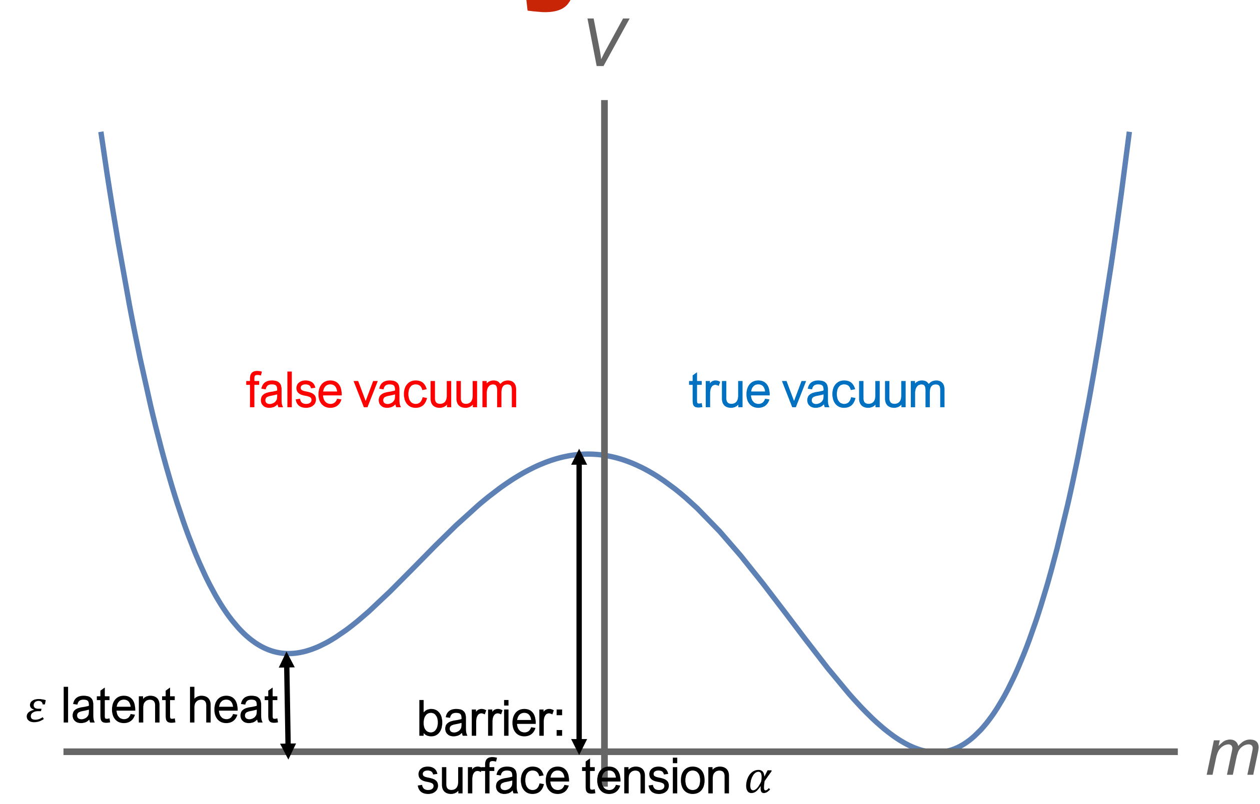
Attractive force **Repulsive force**

Expectation: nucleated bubbles of the true vacuum expand

Decay of the false vacuum (QFT: Coleman scenario)

Digression: vacuum decay in QFT



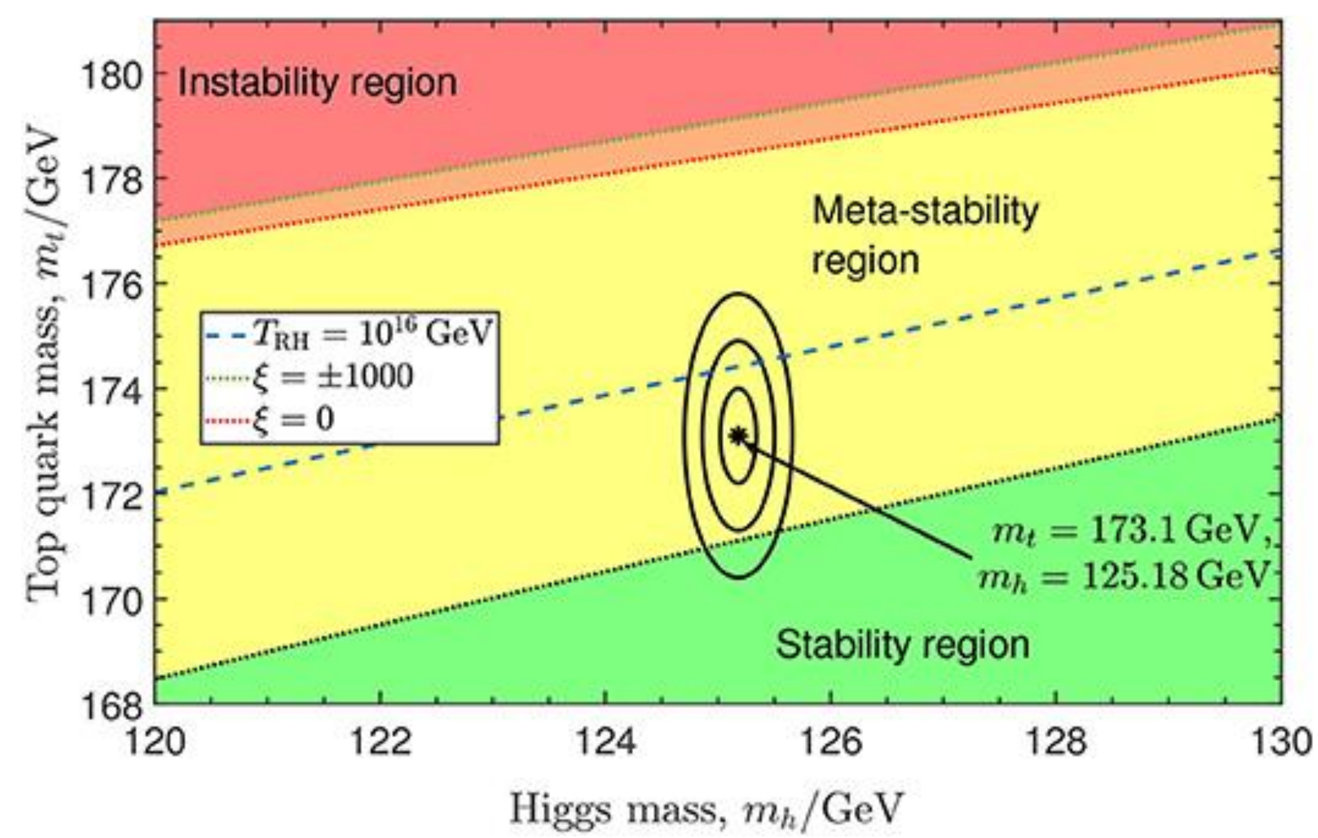


Critical bubble: $\varepsilon V = \alpha A \rightarrow R_*$

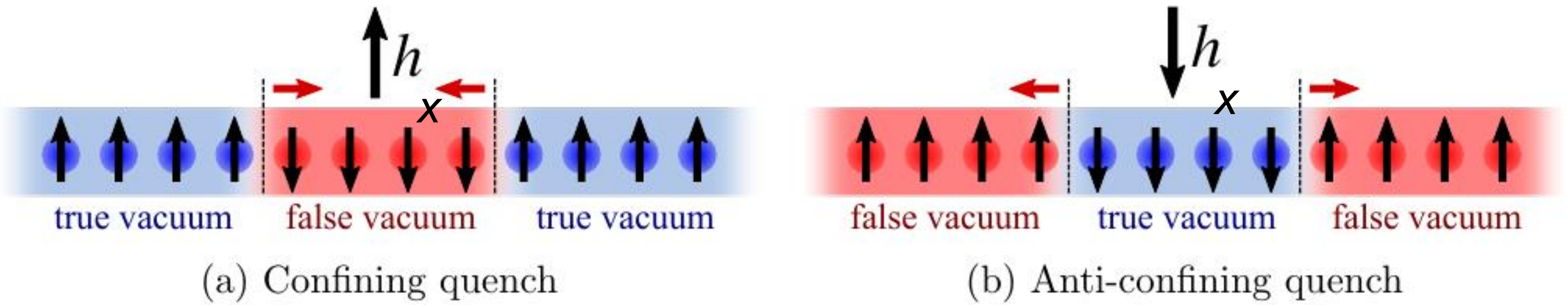
$R < R_*$ **: bubble collapses**

$R > R_*$**: bubble expands**

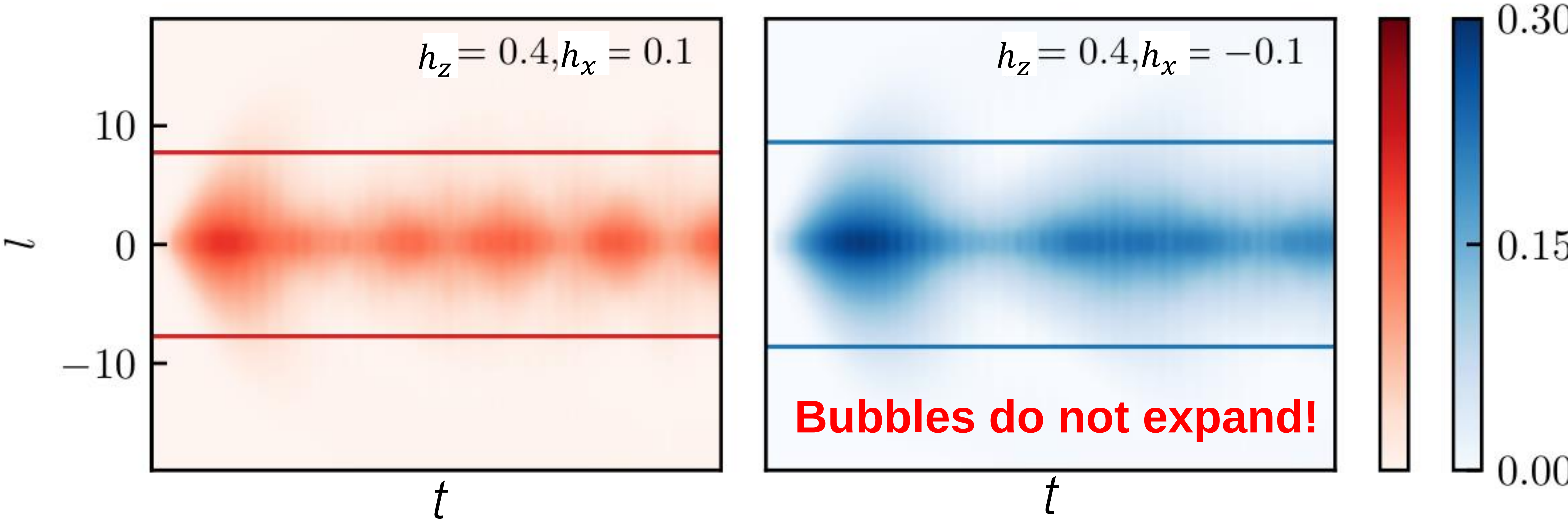
Nucleation rate: given by instanton



Localisation in anti-confining quenches

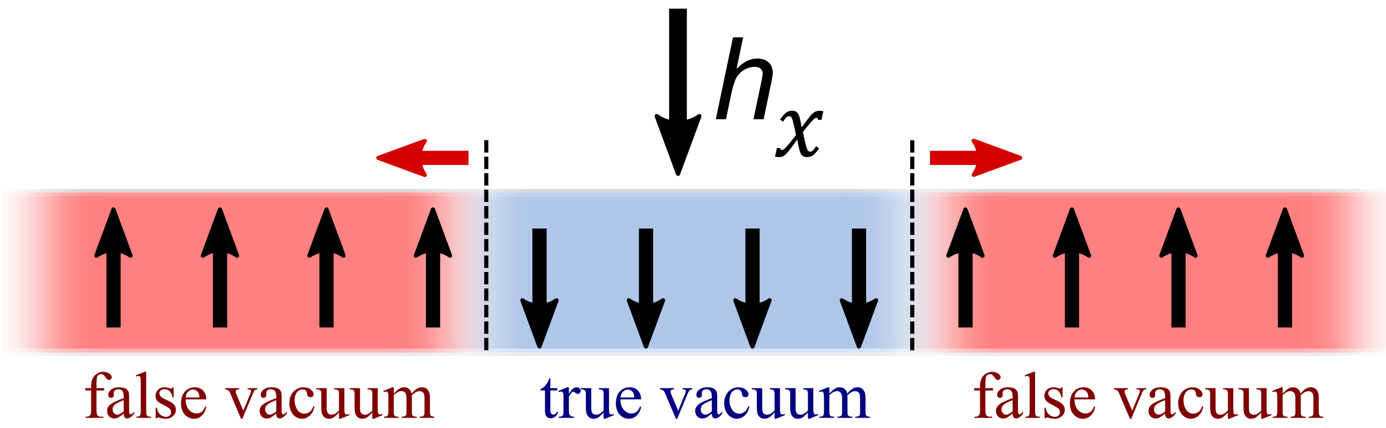


$$\langle \sigma_1^x \sigma_{l+1}^x \rangle_c$$



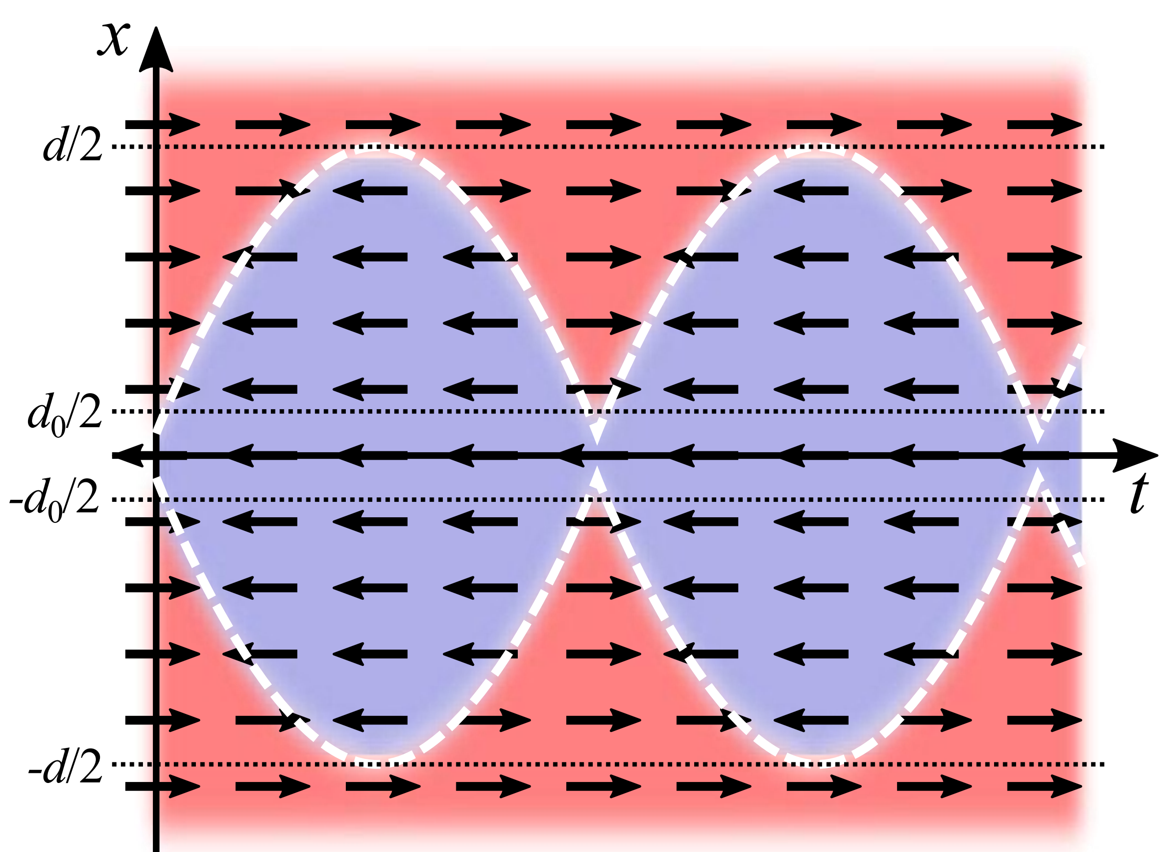
O. Pomponio, M. A. Werner, G. Zaránd and G. Takács,
SciPost Phys. 12, 061 (2022)

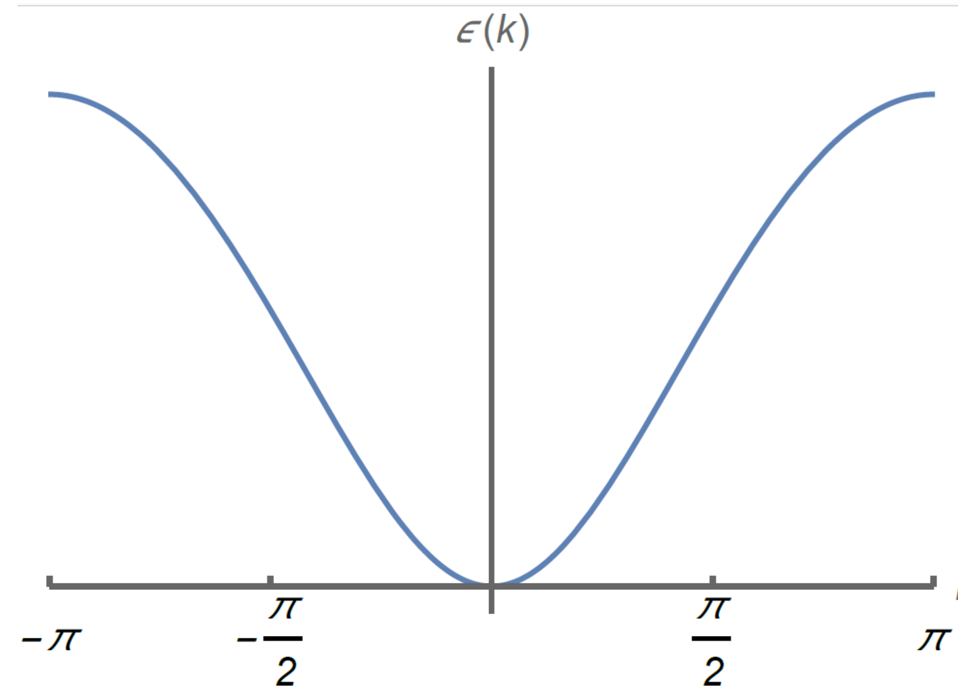
Bloch oscillations



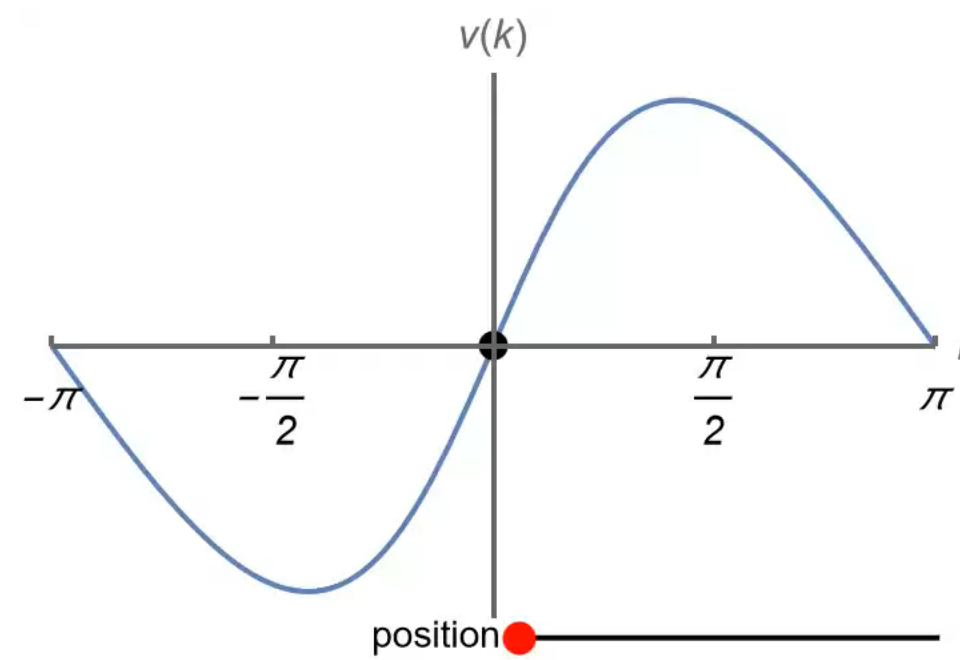
$$V(x) = -\chi|x| \rightarrow \frac{dk}{dt} = \chi$$

$$\omega_{Bloch} = \frac{2\pi}{\chi}$$





$$v(k) = \frac{\partial \epsilon(k)}{\partial k}$$

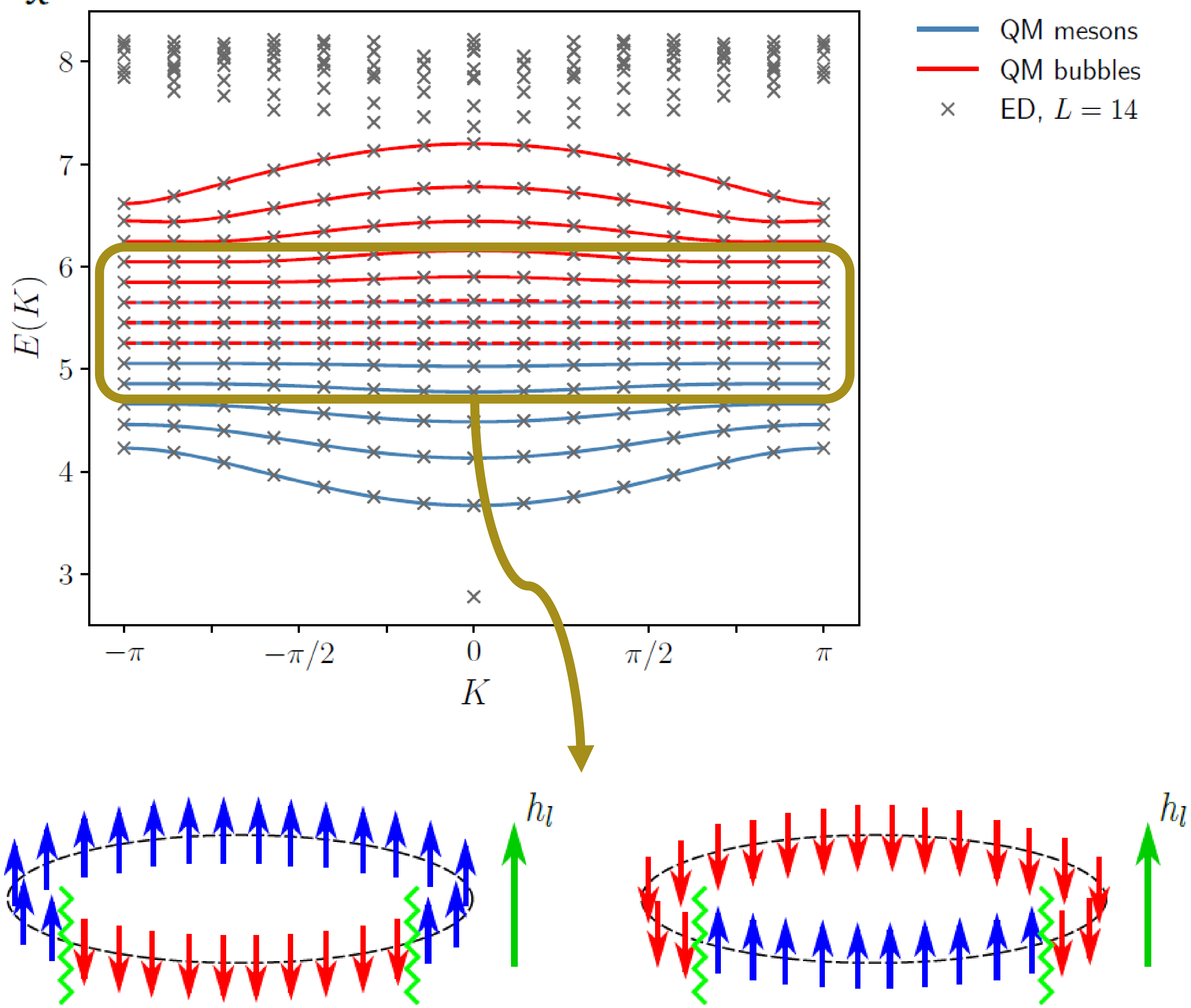


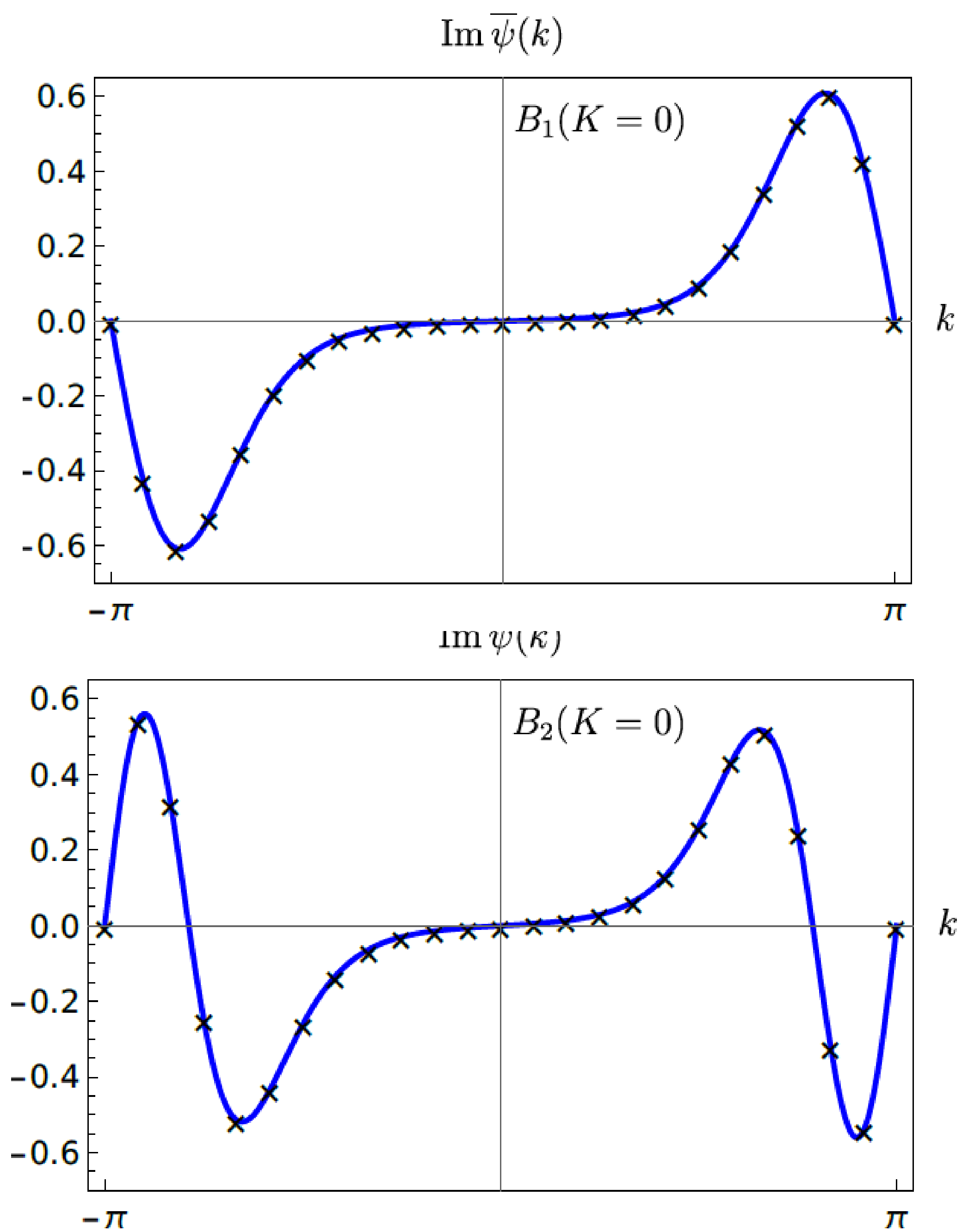
The bubble spectrum

Wannier-Stark localization/Bloch oscillation → localized bubble states

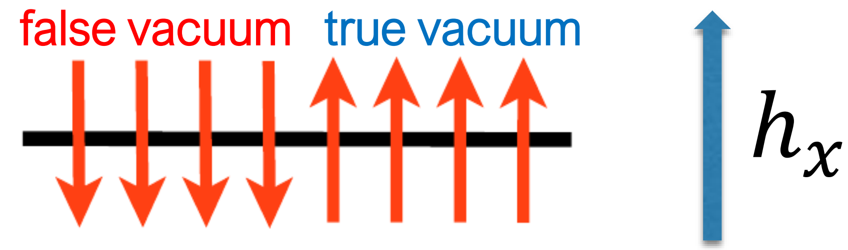
$$\sum_{x'}\left[H_0(x-x',K)-|\chi||x|\right]\overline{\psi}_{n,K}(x')=\overline{E}_n(K)\overline{\psi}_{n,K}(x)$$

Krasznai & Takács, 2024





Local quenches

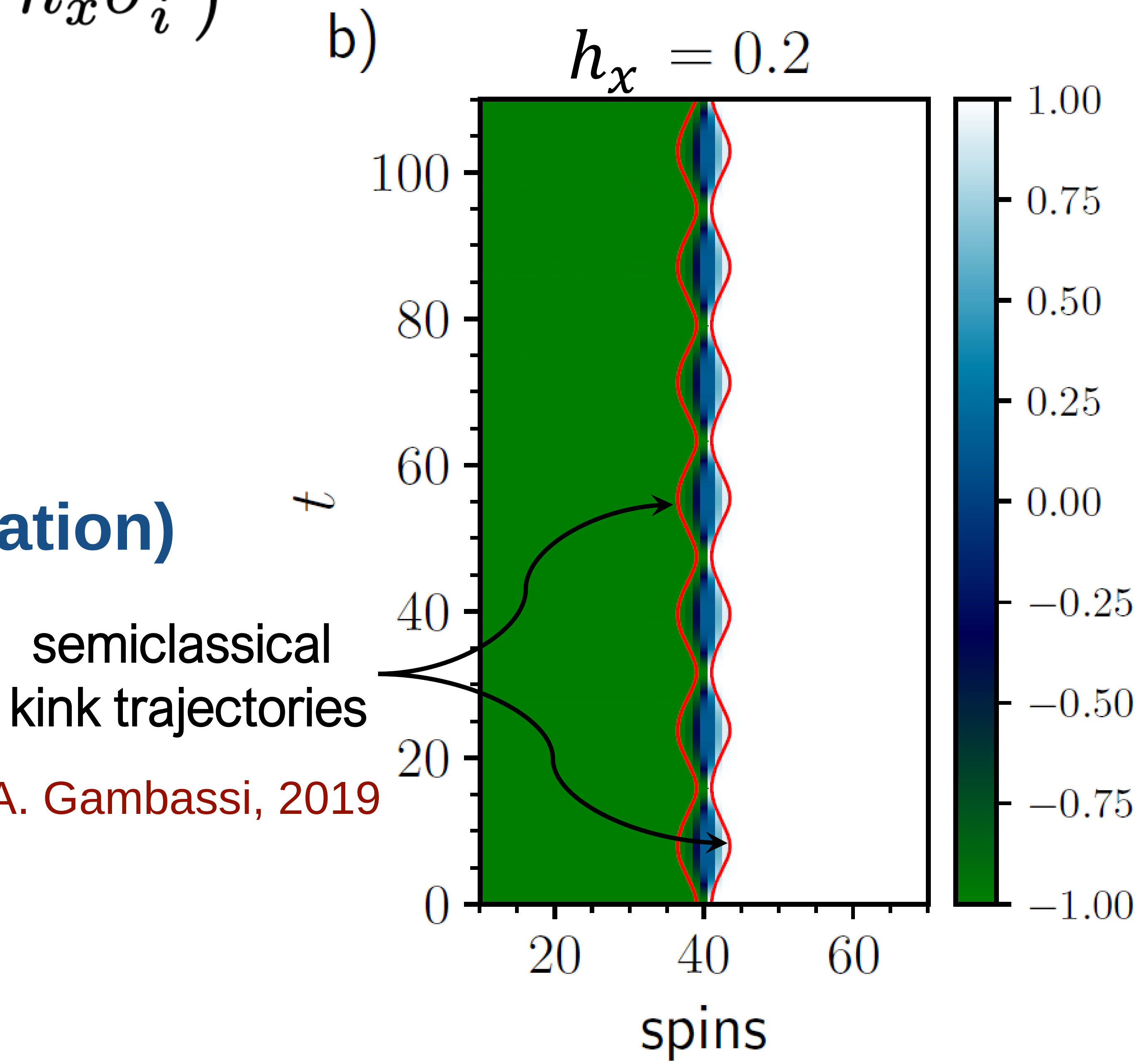


Start system in domain wall initial state

$$H = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$

Right half: confinement

Left half: Bloch oscillations (Wannier-Stark localization)

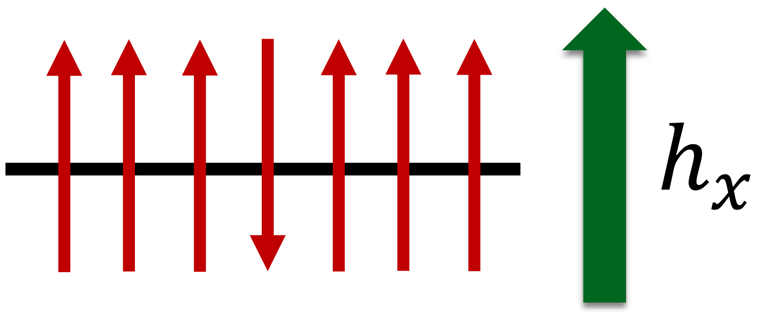


P.P. Mazza, G. Perfetto, A. Lerose, M. Collura, and A. Gambassi, 2019

No surprise here…

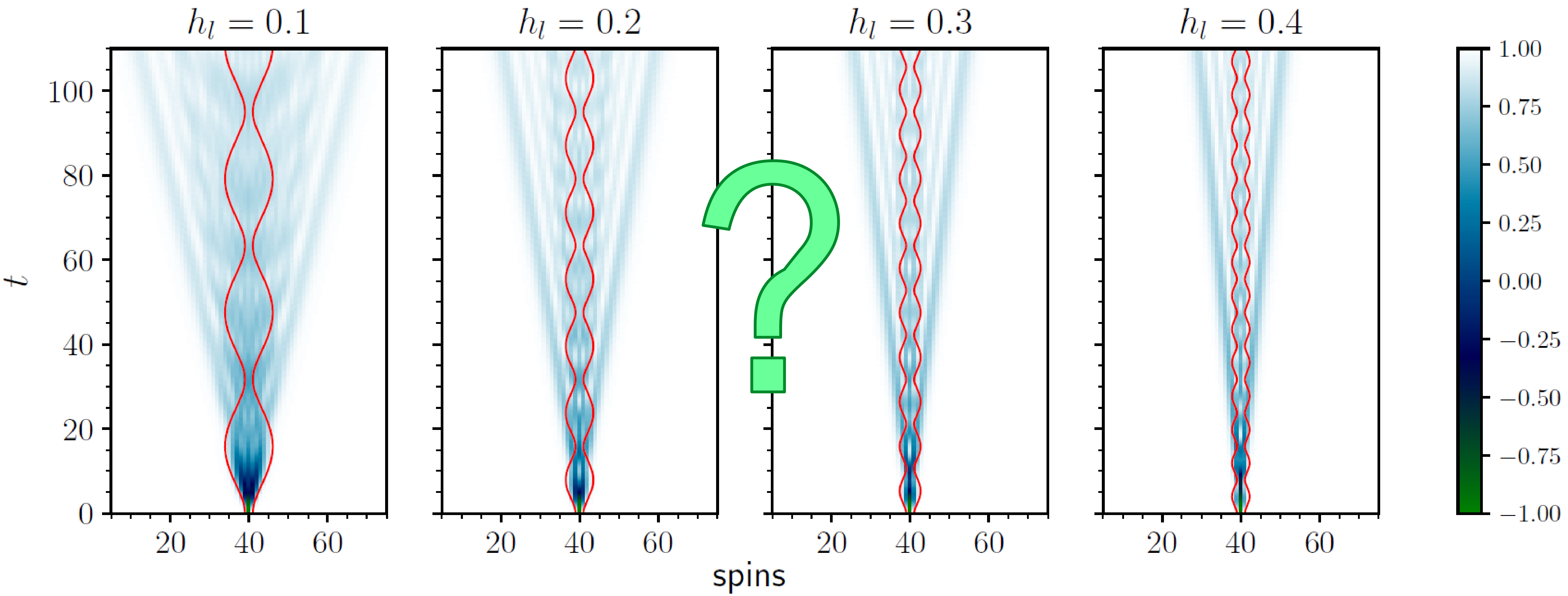
Simulation:
A. Krasznai and G. Takács, 2024

Escaping fronts



Start system in spin-flip initial state

$$H = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$

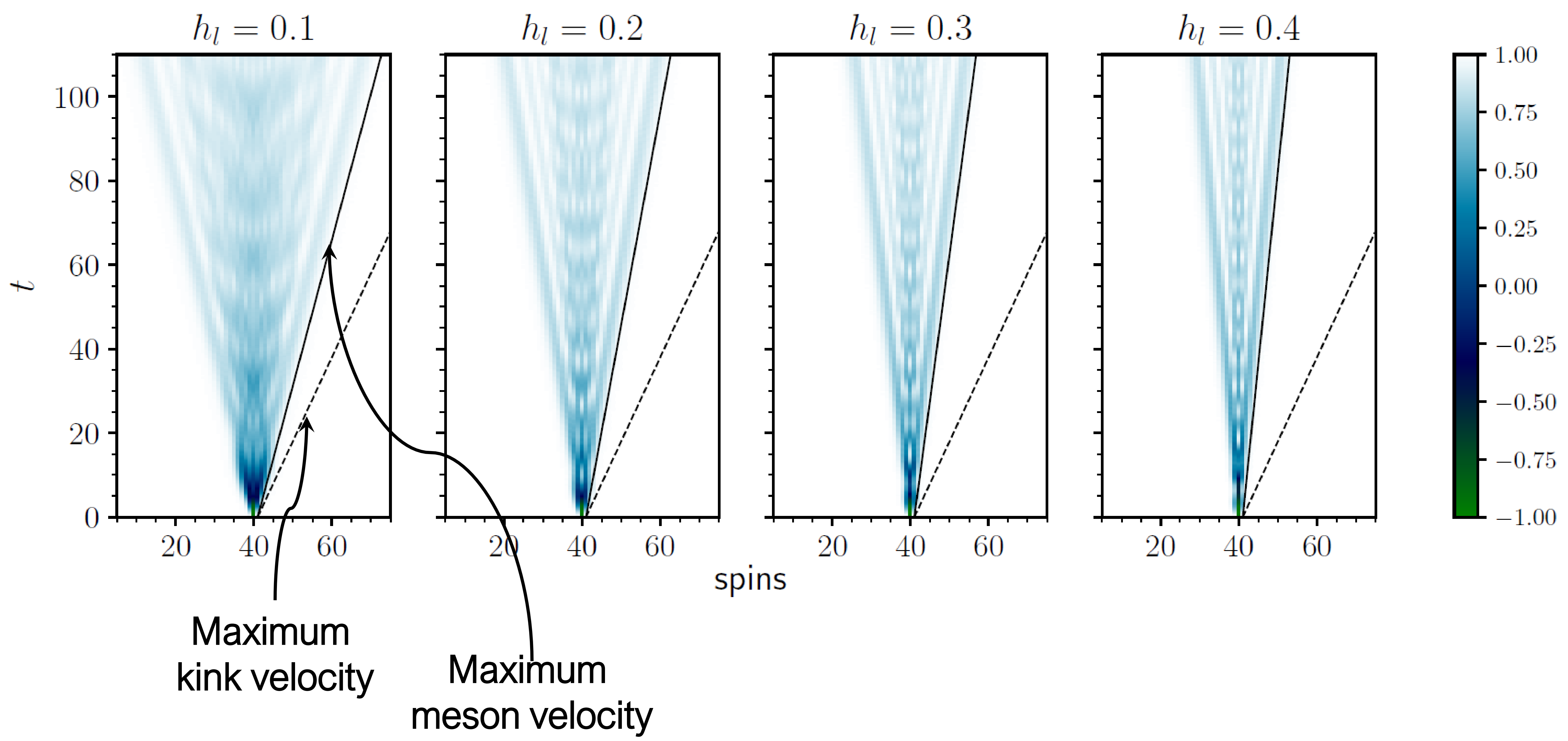


A. Krasznai and G. Takács, 2024

Schrödinger kittens escape confinement

Combining analytic and numerical methods:

Escaping fronts are superpositions of left/right moving single mesons

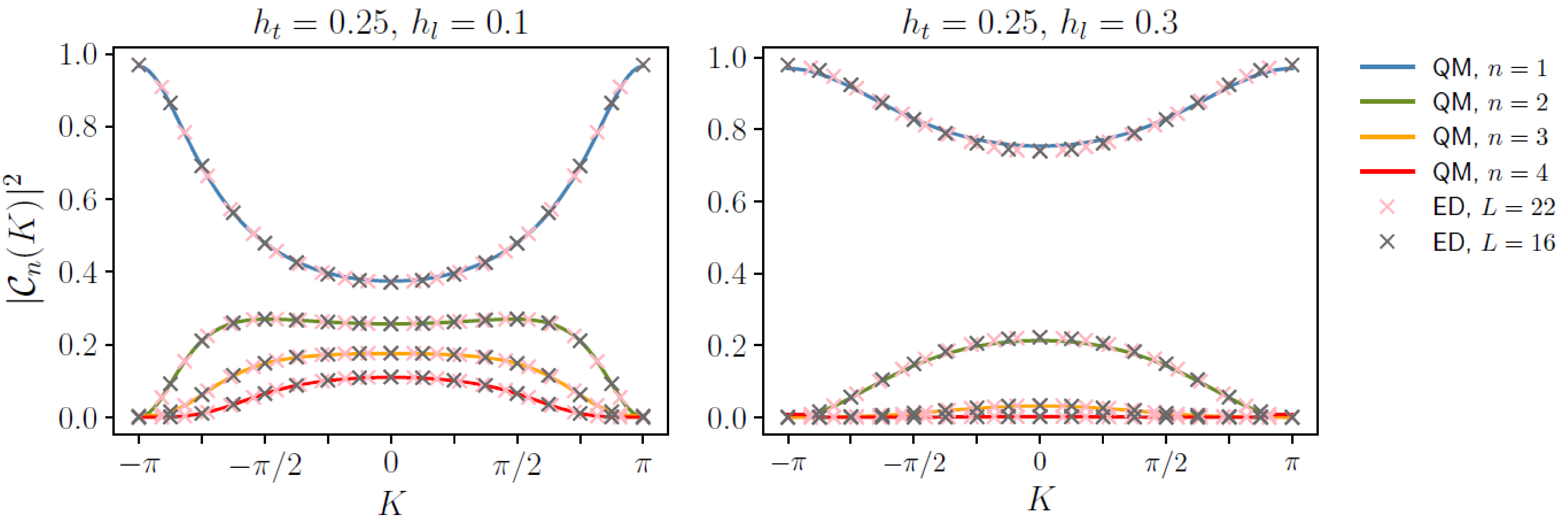


A. Krasznai and G. Takács, 2024

Overlaps of mesons with initial state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{L}} \sum_{n,K} \mathcal{C}_n(K)\,|M_n(K)\rangle$$

The overlaps $\mathcal{C}_n(K)$ can be computed using Jordan-Wigner transformation + meson wave function from Schrödinger equation



A. Krasznai and G. Takács, 2024

Schrödinger kittens escape confinement

Global quench:

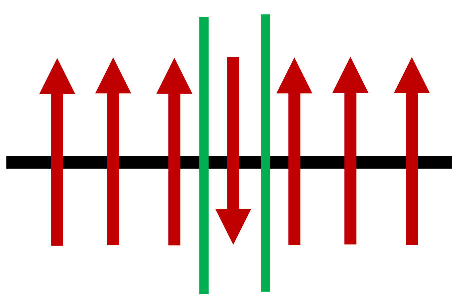
translational invariance only allows to create moving mesons in opposite momentum pairs
- energy threshold!

-> Escaping fronts are strongly suppressed by small probability of tunneling (string breaking/Schwinger effect)

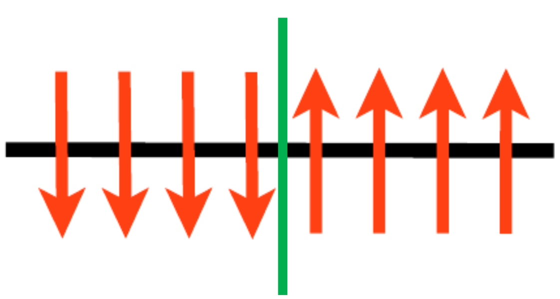
Spin-flip quench:

Single mesons can be created

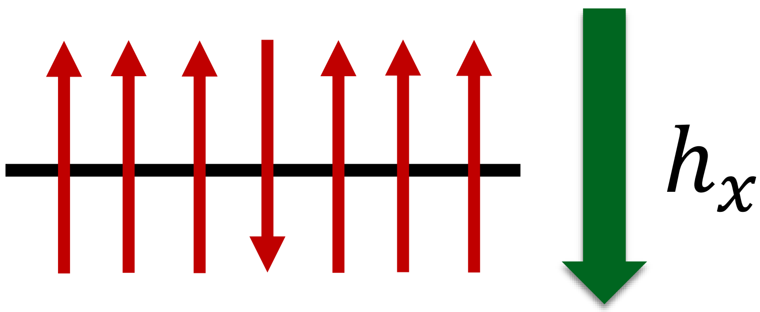
No suppression: locally available energy from spin-flip



Domain wall quench: not enough energy to create a meson

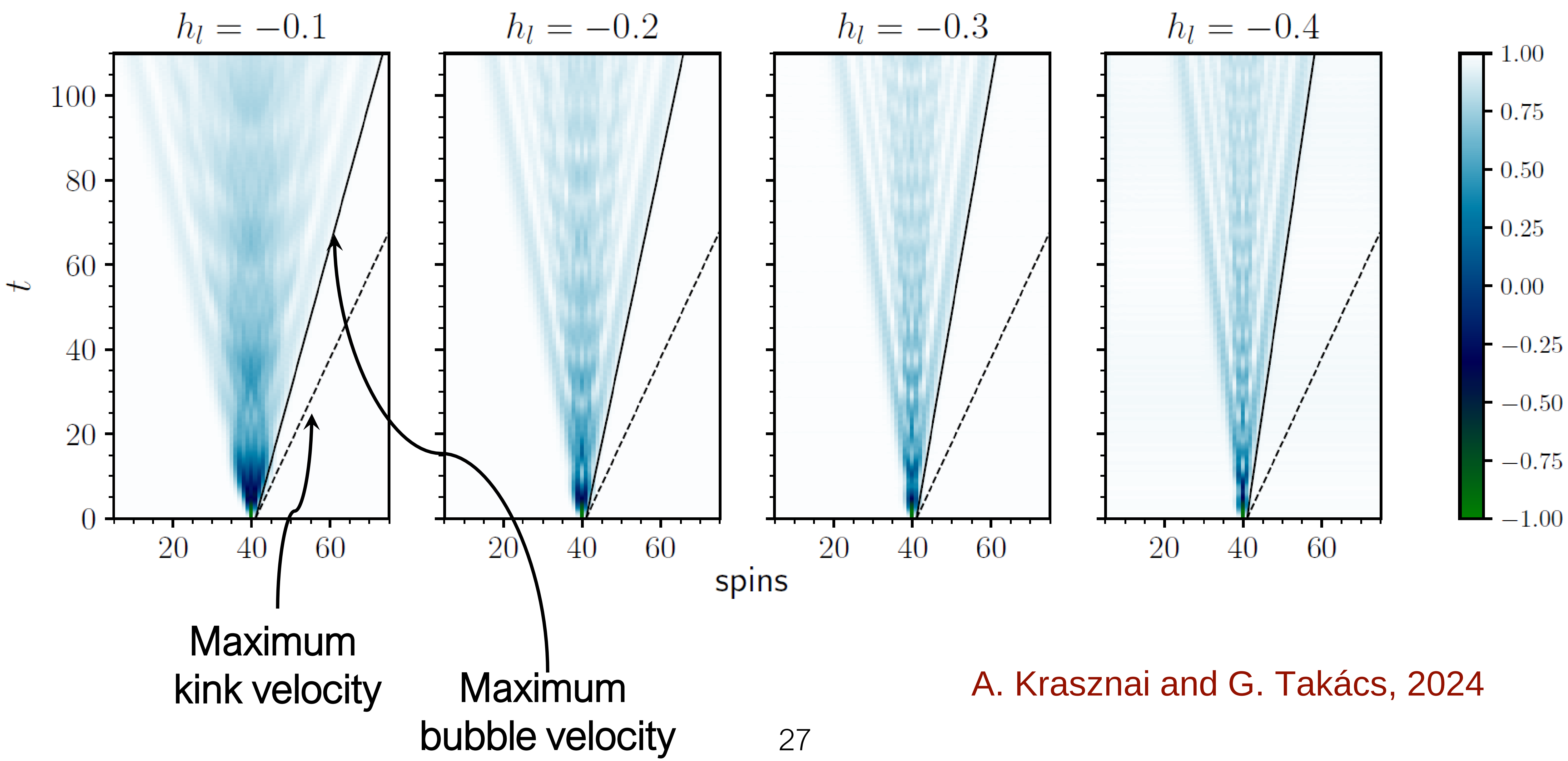


Local quenches induced by spin-flip over the false vacuum



Global quenches: fronts suppressed by Wannier-Stark localization (Bloch oscillations)

Escaping fronts in local quenches: superpositions of left/right moving single, nucleated true vacuum bubbles



A. Krasznai and G. Takács, 2024

Summary

- **Thermalisation of quantum systems is nontrivial**
- **Quantum quench is a paradigmatic, experimentally feasible protocol to study non-equilibrium dynamics**
- **Confinement strongly alters dynamics, suppressing light cone**
- **False vacuum decay can be suppressed by Bloch oscillations**
- **Wannier-Stark localization provides another mechanism to suppress light cone**
- **But: in local quenches, Schrödinger kittens can escape confinement / Wannier-Stark localisation**

Outlook

1. **Confinement alters dynamics in many other systems (including 1+1D QCD, 2d transverse Ising model etc.)**

2. **Experimental realizations**

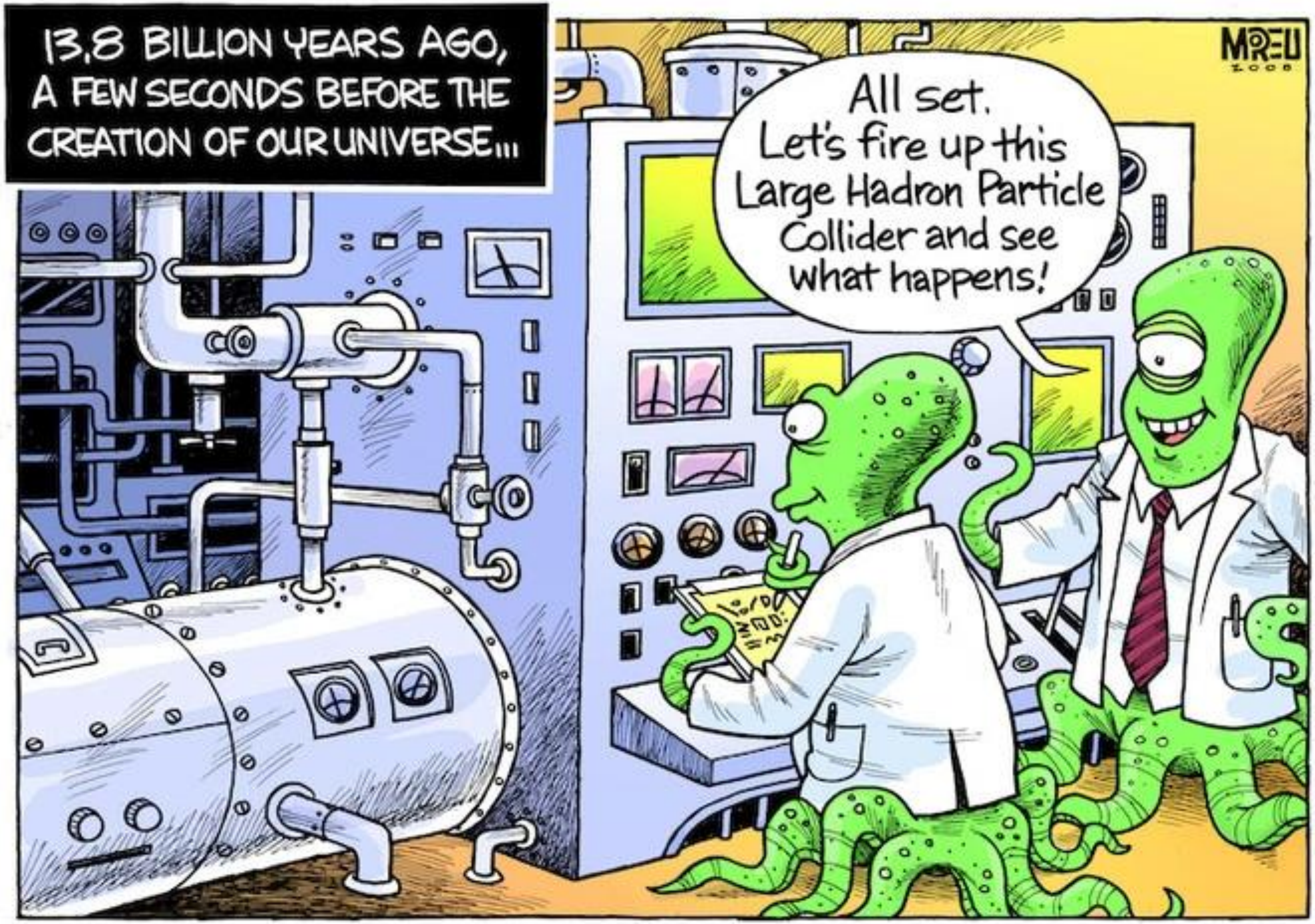
   Confinement: Rydberg atoms
   Quantum simulations

   Vacuum decay: fermionic superfluids

3. **Connection to high energy physics**

   Meta-stability of vacuum can be detected by local quenches by difference between meson and bubble spectra!

F. Wilczek et al., 2023


13.8 BILLION YEARS AGO, A FEW SECONDS BEFORE THE CREATION OF OUR UNIVERSE...
All set. Let's fire up this Large Hadron Particle Collider and see what happens!
MOREU 2008

The end