Four Dimensional Compactifications of Heterotic Theories

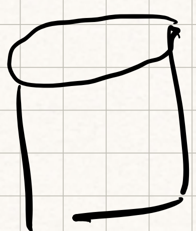
Goal:
* context for NLSM, GLSMs in the study of string vacua.
* Describe/Sketch how to find a 4d EFT given a string vacuum.

"Solutions of string theory" that realize $\mathbb{R}^{3,1}$.

Embedding of the worldvolume of a

string in a target space. Topologically the w.v. is a cylinder.

Study dynamics of the w.v. by a QFT in two-dimensions.



$X(z)$

$\longrightarrow$ target space

This QFT is a Non-linear sigma model (NLSM) w/ some symmetries, e.g. Supersymmetry, conformal invariance + vanishing central charge

$$S = \frac{1}{2\pi\alpha'} \int_{\text{w.v}} d^2 z \Big\{ \big(\underbrace{G_{\mu\nu}(X)}_{\text{metric on the target space}} + \overbrace{B_{\mu\nu}(X)}^{\text{2-form on target space}} \big) \partial X^{\mu} \bar{\partial} X^{\nu} + \dots \Big\},$$

Φ = dilaton

* $G_{\mu\nu}$, $B_{\mu\nu}$, Φ data that defines the NLSM.

* $\partial = \frac{\partial}{\partial z}$, $\bar{\partial} = \frac{\partial}{\partial \bar{z}}$

Analytically continue $t \to i\tau$, identify $\tau = \pm\infty$, compactify those points

 $+ \{\infty\}$ =  or S^2, $\mathbb{P}^1$.

 $\to$ target space.

* $G_{\mu\nu} = \eta_{\mu\nu}$ flat and $B_{\mu\nu} = 0$

$\mathbb{R}^{1,d}$ = flat Lorentzian spacetime.

$G_{\mu\nu}(X)$ nontrivial $\to$ theory has nonlinear kinetic terms.

* Diff + Weyl invariance $\rightarrow$ preserved once quantised.

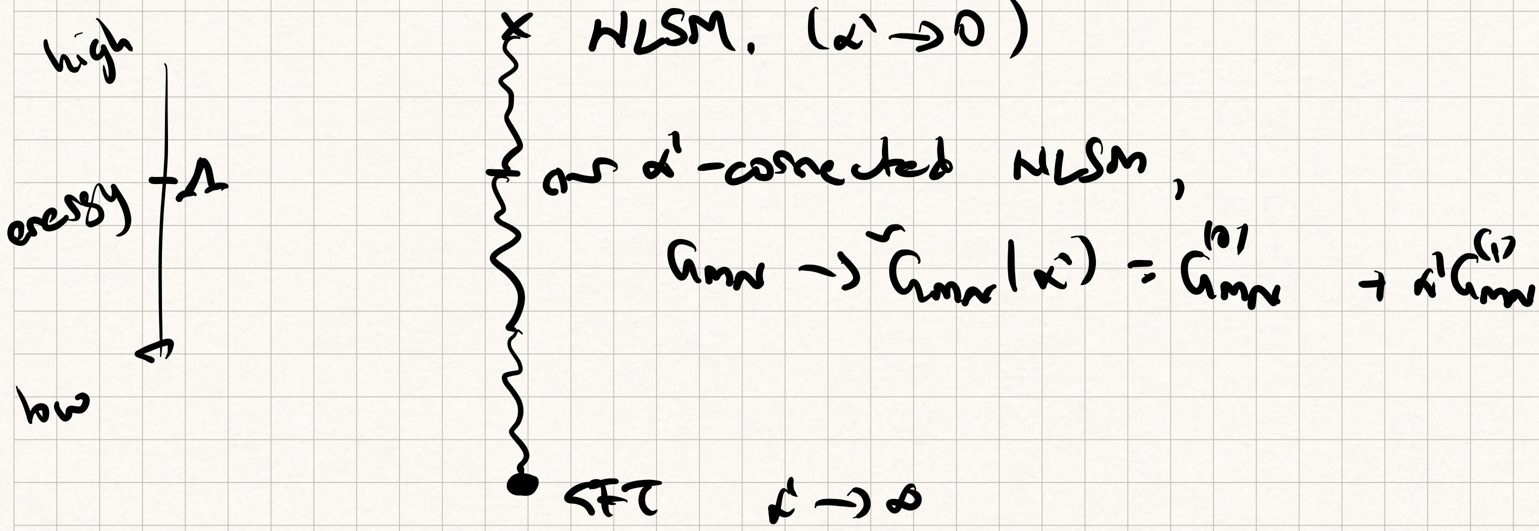
Bosonic string 26 dimensions; NLSM w/ SUSY need 10 dimensions.

* NLSM once quantised induces RG flow.

Coupling constant $\alpha' \sim l_s^2$

$\alpha' \to 0 \iff$ string scale small compared to any curvature scales in target space

$\iff$ NLSM is asymptotically free, weakly coupled, the NLSM at high energies.

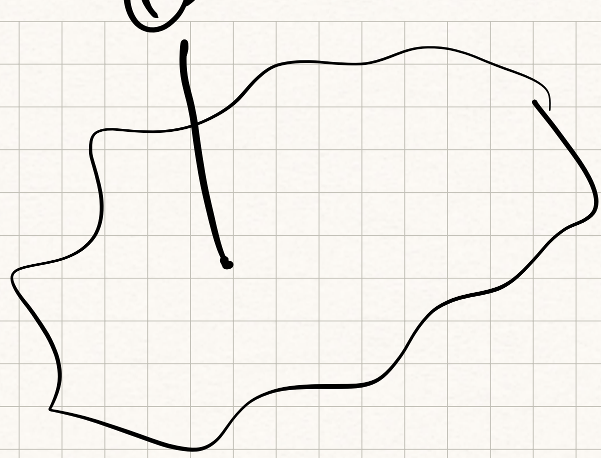


<u>Aside:</u> For heterotic theories $\mathbb{R}^{3,1} \times X$, the couplings have been computed upto and including α'^2.

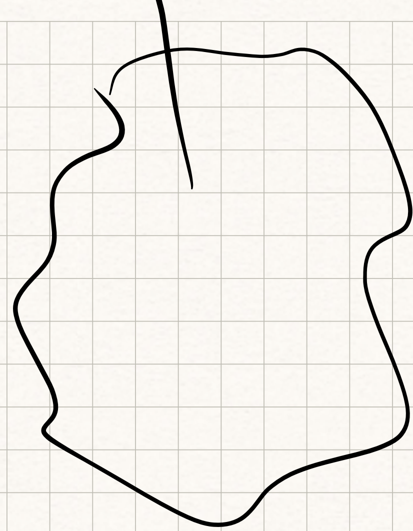
For "nice" NLSMs generate a CFT.

For generic NLSMs RG flow generates a trivial CFT.

- Studying CFTs directly hard. Action principal? Spectrum?



$\overset{?}{\iff}$



$\mathcal{M}_{\text{NLSM}}$

$\mathcal{M}_{\text{CFTs}}$

$\uparrow$

space of string theory vacua.

* In the nbhd of $\alpha' \to 0$ we can study the RG flow via Feynman Diagram.

$$\beta(G_{\mu\nu}, B_{\mu\nu}, \Phi, \dots) = 0 \iff \text{conformal invariance}$$

$$\beta^{1\text{-loop}} = 0 \Rightarrow$$

$$\beta^{2\text{-loop}}$$

$\beta = 0$

$\beta^{3\text{-loop}} = 0.$

$\mathbb{R}^{3,1} \times X$ or CY manifold w/ $\begin{cases} G_{mn} \text{ Ricci flat} \\ dB = 0 \\ \Phi \text{ constant} \end{cases}$

$\beta^{4\text{-loop}} \neq 0.$ $\Rightarrow$ NLSM fails to be conformally invariant at 4-loops $\Rightarrow$ α'^3.

$$\tilde{G}_{mn} = \overset{0}{G}_{mn} + \frac{\alpha'^3}{L^6} G^{(3)}_{mn} + \ldots$$

$\uparrow$

Ricci flat

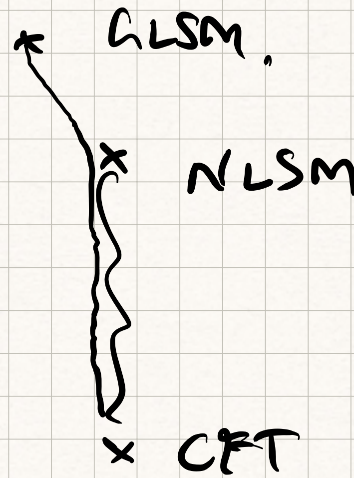
$$\alpha' = l_s^2 = L^2.$$

$$\frac{1}{2\pi\alpha'}\int d^2z\, \partial X \bar{\partial} X$$

* NLSMs are hard!. $\int \{ G_{mn}(X) + B_{mn}(X) \} \partial X^m \bar{\partial} X^n + \ldots$

GLSMs are easier. Linearisation of the KEs.

by introducing a U(1) gauge field



More techniques to study q.c's for GLSMs

and the underlying CFTs.

<u>Heterotic theories</u> $\mathbb{R}^{3,1} \times X$ + gauge field.

<u>Data:</u> G_{mn}, B_{mn}, Φ, gauge bundle E w/ a

connection A.

Defines a NLSM which is chiral. So the

left moving and right moving sectors are different.

Heterotic theories that realise $\mathbb{R}^{3,1}$ w/ $N=1$ supersymmetry in $\mathbb{R}^{3,1}$ are described NLSMs w/ supersymmetry. (0,2).

Compare w/ type II string vacua that realise $\mathbb{R}^{3,1}$, NLSMs w/ (2,2). Often studied in the context CY and mirror symmetry.

(2,2) theories have A- and B-twists which restrict to sectors of the moduli space.

(0,2) theories do not have A, B twists. There are quasitopological twists A/2- B/2-twists.

(0,2) GLSM
A/2-twist

quantities

(0,2) NLSM

invaria… along the RG flow

$\times$ (0,2) CFT w/ A/2-twists.

Other tools to study space of string solutions.

$$\beta^{\text{1-loop}} = 0 \iff \text{Spacetime Supergravity being satisfied}$$

<u>Heterotic Theories</u>

$$F = dA + A^2$$

$$\Theta^+ = \Theta^{LC} + \tfrac{1}{2} H$$

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}X\sqrt{g_{10}}\, e^{-2\Phi}\Big\{\mathcal{R} - \frac{1}{2}|H|^2 + 4(\partial\Phi)^2 - \frac{\alpha'}{4}\big(\operatorname{Tr}|F|^2 - \operatorname{Tr}|R(\Theta^+)|^2\big)\Big\}$$

$$H_3 = dB_2 - \frac{\alpha'}{4} CS(A) + \frac{\alpha'}{4} CS(\Theta^+)$$

$$dH_3 = -\frac{\alpha'}{4}\operatorname{tr} F^2 + \frac{\alpha'}{4}\operatorname{tr} R_+$$

EOM

$$\mathcal{R}_{MN} + 2\nabla_M\nabla_N\Phi - \frac{1}{4}H_{MAB}H_N{}^{AB} - \frac{\alpha'}{4}\Big(\operatorname{Tr} F_{MP}F_N{}^P - R_{MPAB}(\Theta^+)R_N{}^{PAB}(\Theta^+)\Big) + \mathcal{O}(\alpha'^3) = 0\,,$$

$$\nabla^M(\mathrm{e}^{-2\Phi}H_{MNP}) + \mathcal{O}(\alpha'^3) = 0\,,$$

$$\mathcal{D}^{\mathrm{B}\,M}(\mathrm{e}^{-2\Phi}F_{MN}) + \mathcal{O}(\alpha'^3) = 0\,,$$

$$\Leftrightarrow \quad \beta^{1\text{-loop}} + \beta^{2\text{-loop}} + \beta^{3\text{-loop}} = 0$$

Supersymmetry: $\mathcal{N}=1$ in $\mathbb{R}^{3,1}$

$$\Leftrightarrow$$

$$
\begin{aligned}
\delta\Psi_M &= \left(\partial_M + \frac{1}{4}\Gamma_{AB}\left(\Theta_M^{-\,AB} + \alpha' P_M{}^{AB}\right) + \mathcal{O}(\alpha'^3)\right)\eta = 0\,,\\
\delta\lambda &= -\frac{1}{2\sqrt{2}}\left(\partial\!\!\!/ - \frac{1}{2}H\!\!\!\!/ + \frac{3}{2}\alpha' P\!\!\!/ + \mathcal{O}(\alpha'^3)\right)\eta = 0\,,\\
\delta\chi &= -\frac{1}{2}F\!\!\!/\,\eta + \mathcal{O}(\alpha'^3) = 0\,.
\end{aligned}
$$

where

$$P_{MAB} = 6e^{2\Phi}\nabla^{(-)N}\left(e^{-2\Phi}\mathrm{d}H\right)_{MNAB}\,.$$

$$\Leftrightarrow$$

If we work to $\mathcal{O}(\alpha')$ (ie first order in α') then

- X is a complex manifold w/ J its complex structure. J tells you what is holomorphic.
- $H = d^c\omega$, $\omega = i g_{\mu\bar\nu}\, dx^{\mu\bar\nu} = J_m{}^p g_{pn}\, dx^{mn}$.

$$= \frac{1}{3!} J_n{}^m J_p{}^q J_s{}^t (d\omega)_{mqt}\, dx^{nps}$$

$$\left(dx^{nps} = dx^n \wedge dx^p \wedge dx^s. \right.$$

For fixed J, $J = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$,

$$H = i(\partial - \bar{\partial})\omega \qquad \partial = dx^\mu \partial_\mu .$$

- $F = dA + A^2$, $\quad F = \underbrace{\cancel{F^{(0,2)}}}_{0} + F^{(1,1)} + \underbrace{\cancel{F^{(2,0)}}}_{0}$

$$F^{(0,2)} = 0 \iff E \to X \text{ holomorphic bundle}$$

- $g^{\mu\bar{\nu}} F_{\mu\bar{\nu}} = 0$ Hermitian Yang Mills Eqn.

- $c_1(X) = 0$.

$$dH = -\frac{\alpha'}{4} \operatorname{tr} F^2 + \frac{\alpha'}{4} \operatorname{tr} R^2 \neq 0$$

$$= 2i\, \bar{\partial}\partial\omega$$

So $H \neq 0 \Rightarrow \bar{\partial}\omega \neq 0$

<u>Remember</u>: all of these come from $\beta^{3\text{-loop}} = 0$

$\alpha' \to 0$.

Field of Deformation theory of the non-kähler manifolds w/ bundles, w/ Supersymmetry + Bianchi identities