MATRIX

Quantum Curves, Integrability and Cluster Algebras

13 – 18 December 2021

Talk Titles and Abstracts

Monday, 13 December 2021

Huang

Title: Welcome to super hyperbolic surface theory!

Abstract: We give a friendly and practical introduction to the theory of super hyperbolic surfaces, laying the groundwork for the super Fuchsian representation theory of marked super hyperbolic surfaces.

Giacchetto

Title: Hurwitz theory, with (a) spin

Abstract: Spin Hurwitz numbers enumerate branched Riemann covers weighted by the parity of theta characteristics. They can be realised as vacuum expectation values on the Fock space of type B, and they obey the BKP integrable hierarchy. We prove that topological recursion for these numbers is equivalent to an ELSV-type formula expressing spin Hurwitz numbers as integrals on the moduli space of curves of an explicit product between Witten's class and Chiodo's class. The topological recursion conjecture has recently been proved by Alexandrov and Shadrin in a more general framework for BKP integrability. Time permitting, I will explain possible applications towards the spin Gromov–Witten/Hurwitz correspondence. This is based on joint work with R. Kramer and D. Lewański.

Tuesday, 14 December 2021

Bouchard

Title: Topological Recursion, Quantum Curves, and Atlantes Hurwitz Numbers

Abstract: The TR/QC correspondence states that the CEO topological recursion (and its higher analog), which associates a sequence of differentials to a spectral curve, can be used to reconstruct the WKB asymptotic solution of a differential equation that is a quantization of the spectral curve (known as a "quantum curve"). In this work we prove the TR/QC correspondence for a class of spectral curves with essential singularities, which can be obtained as limits of sequences of algebraic spectral curves. To this end, we must generalize topological recursion, to include contributions from the essential singularities. This generalization is in fact natural, since it is needed for the topological recursion to be consistent with limits of sequences of algebraic curves. The prototypical example is the spectral curve which is known to give rise to r-spin Hurwitz numbers via the usual topological recursion; we show that, for the same spectral curve, our natural generalization of the topological recursion instead computes atlantes Hurwitz numbers, and reconstructs the WKB solution of the appropriate quantum curve. This is particularly interesting given that atlantes Hurwitz numbers had so far evaded topological recursion methods.

This is joint work with Reinier Kramer and Quinten Weller.

Buryak

Title: Counting meromorphic differentials on the Riemann sphere and the KP hierarchy

Abstract: There is now a large family of results saying that the generating series of Hurwitz numbers of various sorts gives a solution of the KP hierarchy. This is one of manifestations of a deep relation between curve counting invariants and integrable systems. In our joint work with Paolo Rossi and Dimitri Zvonkine we observed a new phenomenon in this framework. We studied the numbers of residueless meromophic differential on the Riemann sphere and proved that these numbers are exactly the coefficients of the equations of the dispersionless KP hierarchy. If time permits, I will also briefly explain the geometric interpretation of the equations of the full KP hierarchy. For this, one has to consider the full partial cohomological field theory given by the classes of residueless meromophic differentials in the moduli spaces of stable curves and the associated intersection numbers.

MATRIX

Wednesday, 15 December 2021

Ovenhouse

Title: Double Dimers and the Super Ptolemy Relation

Abstract:

Given a quadrilateral inscribed in a circle, Ptolemy's theorem relates the lengths of the diagonals to the lengths of the sides. Given an inscribed polygon, the Ptolemy relation can be used to express the length of any diagonal as a Laurent polynomial in the lengths of the diagonals in some fixed triangulation. This is a realization of type-A cluster algebras, where the cluster variables are the lengths of the diagonals. There are formulas (due to Musiker and Schiffler) for these Laurent polynomials in terms of dimer covers (perfect matchings) of a certain graph.

Penner and Zeitlin have recently defined a super algebra which generalizes this situation by introducing new non-commuting variables. They also defined a super version of Ptolemy's relation. In joint work with Musiker and Zhang, we gave a formula for the corresponding elements of the super algebra in terms of double dimer covers on the same graph.

Alexandrov

Title: KP integrability of triple Hodge integrals

Abstract:

Enumerative geometry invariants are closely related to integrable systems. In particular, unexpectedly often the generating functions of the enumeration geometry invariants are tau-functions of integrable hierarchies of KP type. In my talk, I will describe the identification of the generating function of the triple Hodge integrals satisfying the Calabi-Yau condition with a tau-function of the KP hierarchy. In this identification, an infinite-dimensional group acting on the space of the solutions of the KP hierarchy plays an important role. In the introductory part of the talk, I will explain some basic properties of the KP tau-functions and corresponding infinite-dimensional group.

Thursday, 16 December 2021

Schrader

Title: Bifundamental Baxter operators

Abstract: The Hamiltonians of the GL(n) open q-difference Toda lattice as well as their eigenfunctions, the q-Whittaker functions, can be neatly described by means of the system's Baxter operator. The fact that the Baxter operator acts diagonally in the basis of Whittaker functions can be regarded as a q-deformed, continuous analog of the generating function of all Pieri rules for Schur functions. In joint work with Alexander Shapiro we introduce a kind of GL(n) x GL(m)-generalization of this Baxter operator, such that the original Baxter operator corresponds to the case m=1. These new operators can be realized as quantum cluster transformations, and govern the cluster structure on K-theoretic Coulomb branches of 3d N=4 gauge theories with bifundamental matter. This perspective leads to a categorification of the bifundamental Baxter operators in which cluster mutations are promoted to exact triangles in the derived category of coherent sheaves on a convolution variety.

Garcia

Title: Quantisation of spectral curves of arbitrary rank and genus via topological recursion

Abstract: The topological recursion is a ubiquitous procedure that associates to some initial data called spectral curve, consisting of a Riemann surface and some extra data, a doubly indexed family of differentials on the curve, which often encode some enumerative geometric information, such as volumes of moduli spaces, intersection numbers and knot invariants. The quantum curve conjecture claims that one can associate to a spectral curve a differential equation, whose solution can be reconstructed by the topological recursion applied to the original spectral curve. I will explain how starting from loop equations, one can construct a system of KZ equations whose solutions are vectors of wave functions built from topological recursion. These equations can often be interpreted as PDEs with respect to the moduli of the spectral curves. I will explain the idea to obtain an associated Lax pair that shares the same pole structure as the initial spectral curve, which in particular solves the conjecture affirmatively for a large class of spectral curves. I will comment on the technicalities that arise when attacking this conjecture for generic algebraic spectral curves, the solutions we proposed and what remains to be done. This is based on joint work with B. Eynard, in which we treated the hyperelliptic case, and with N. Orantin and O. Marchal, in which we deal with the generalisation to spectral curves of arbitrary rank, albeit with simple ramifications.

MATRIX

Friday, 17 December 2021

Zeitlin

Title: q-opers and the geometric approach to the Bethe ansatz equations

Abstract: I will talk about the notion of a q-oper, which has recently emerged in the context of quantum integrable systems, and, somewhat surprisingly, in enumerative geometry. Originally proposed as a q-deformation of an oper connection on the projective line, it found its first application in the q-deformation of the fundamental example of the geometric Langlands correspondence: the relation between the G-oper connections of the specific type and the spectrum of the Gaudin model for the Langlands dual Lie algebra of ^LG. The analog of the Gaudin model in the q-deformed case is the spin chain model of XXZ type -- the functional relation, known as QQ-system (which leads to the so-called Bethe equations), characterizes its spectrum. Among other applications of the q-oper formalism, I will show how the QQ-systems and their extensions emerge in the context of the relations between generalized minors, thereby putting the Bethe Ansatz equations in the framework of cluster mutations, known in the theory of double Bruhat cells.