Integrable and non-integrable models in quantum optics

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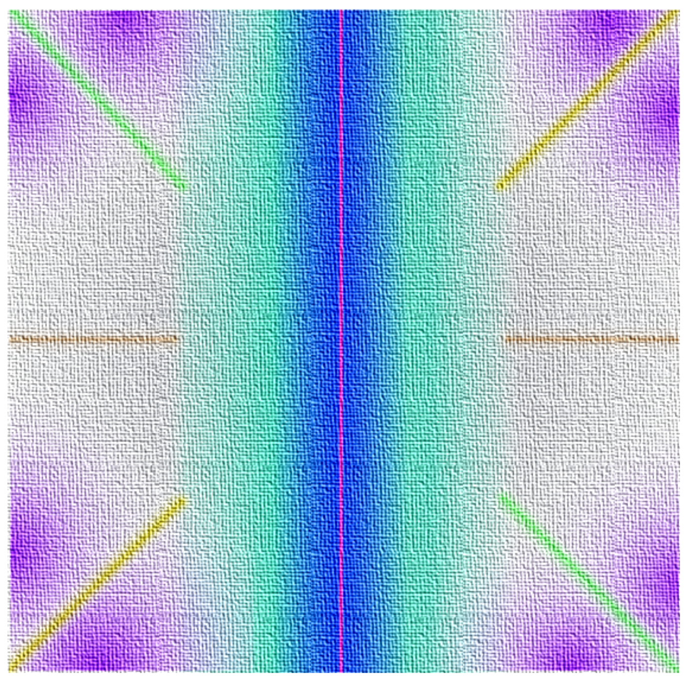

“Integrability in Low-Dimensional Quantum Systems”
University of Melbourne, July 11, 2017

Outline:

- classical and quantum integrability
- quantum optical models
- a criterion for quantum integrability
- solution of the quantum Rabi model
- solution of the two-photon quantum Rabi model
- the non-integrable Dicke model
- conclusions

Integrable systems in classical mechanics:

Liouville: $H(q_i, p_i), i = 1, \ldots, N$

If there exist N functions $L_j(q_i, p_i)$ with $\{L_j, L_k\} = 0, \quad L_1 = H$
the system is integrable

canonical transformation to action-angle variables:

$$(\boldsymbol{q}, \boldsymbol{p}) \longrightarrow (\boldsymbol{I}, \boldsymbol{\phi}) \qquad H(\boldsymbol{q}, \boldsymbol{p}) \longrightarrow \tilde{H}(I_1, \ldots I_f)$$

canonical equations:

$$\dot{\phi}_j = \frac{\partial \tilde{H}}{\partial I_j} = \omega_j \qquad \dot{I}_j = -\frac{\partial \tilde{H}}{\partial \phi_j} = 0$$

Non-integrable systems possible for $N \geq 2$

Is this notion sensible in linear quantum mechanics?

continuous degree of freedom $\dim\mathcal{H} = \infty$

discrete degree of freedom $\dim\mathcal{H} < \infty$

$$N = f_c + f_d$$

no consensus on the definition of integrability in quantum mechanics

Caux, Mossel 2011

Bethe ansatz, Yang-Baxter integrability

Bethe 1931
Yang 1967
Baxter 1973

Berry-Tabor criterion

Berry, Tabor 1977

integrable — Poissonian level statistics

non-integrable — Wigner-Dyson level statistics

only applicable for $f_c \geq 2$

what happens for $f_c = 1$ and $f_d = 1$?

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(almost) simplest case:

$$H_{\mathrm{Qbit}} = \Delta \sigma_z \qquad H_{\mathrm{int}} = g\sigma_x(a^\dagger + a) \qquad H_{\mathrm{rad}} = \omega a^\dagger a$$

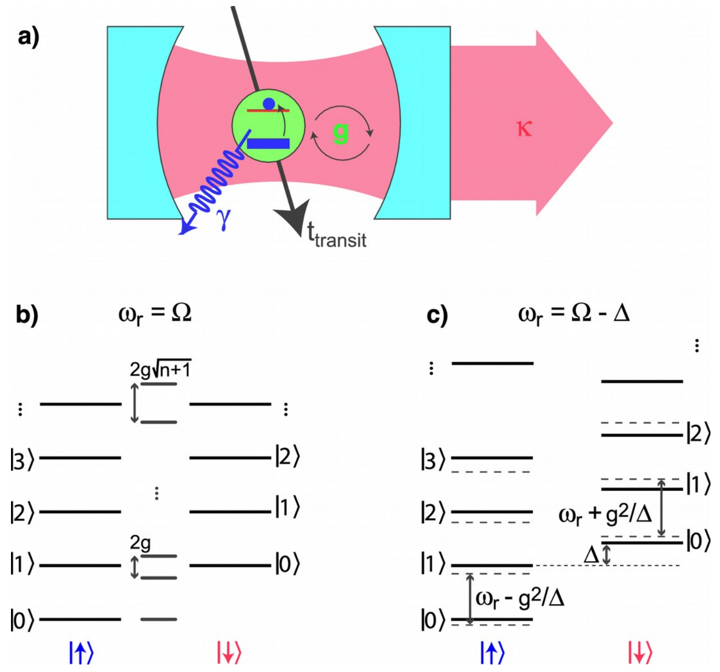
Qbit or two-level atom — dipole coupling — single mode of radiation field

$$H_R = \Delta \sigma_z + g\sigma_x(a^\dagger + a) + \omega a^\dagger a$$

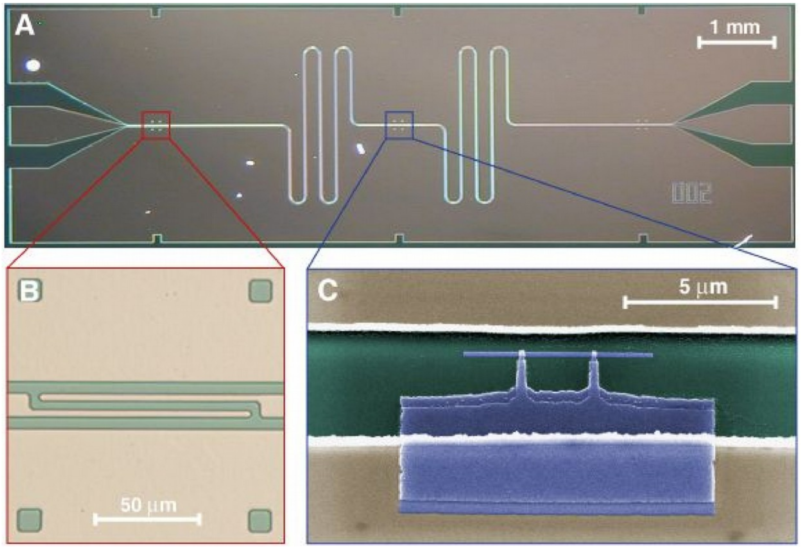
quantum Rabi model (single-mode spin-boson model)

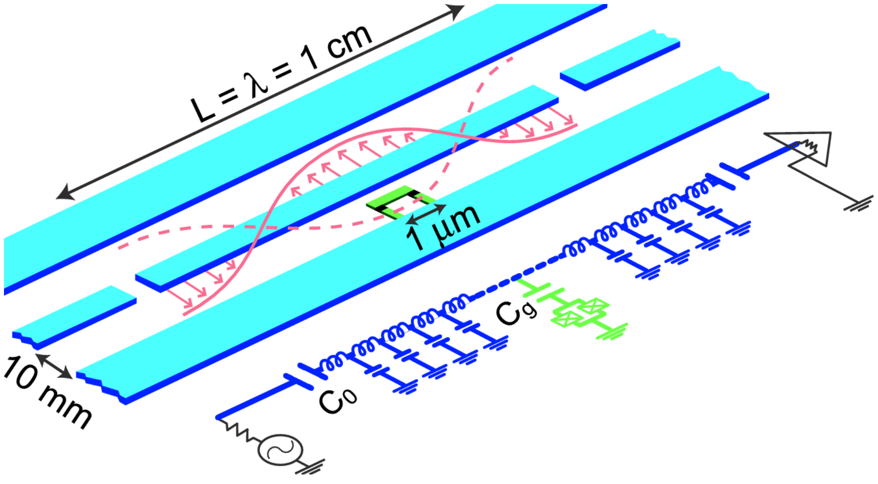
Rabi 1936
Jaynes, Cummings 1963

Cavity QED: few atoms interact with a single mode of the radiation field



implementation of cavity QED within circuit QED:
atoms are replaced by nanoscaled SQUIDS (flux Qbits)





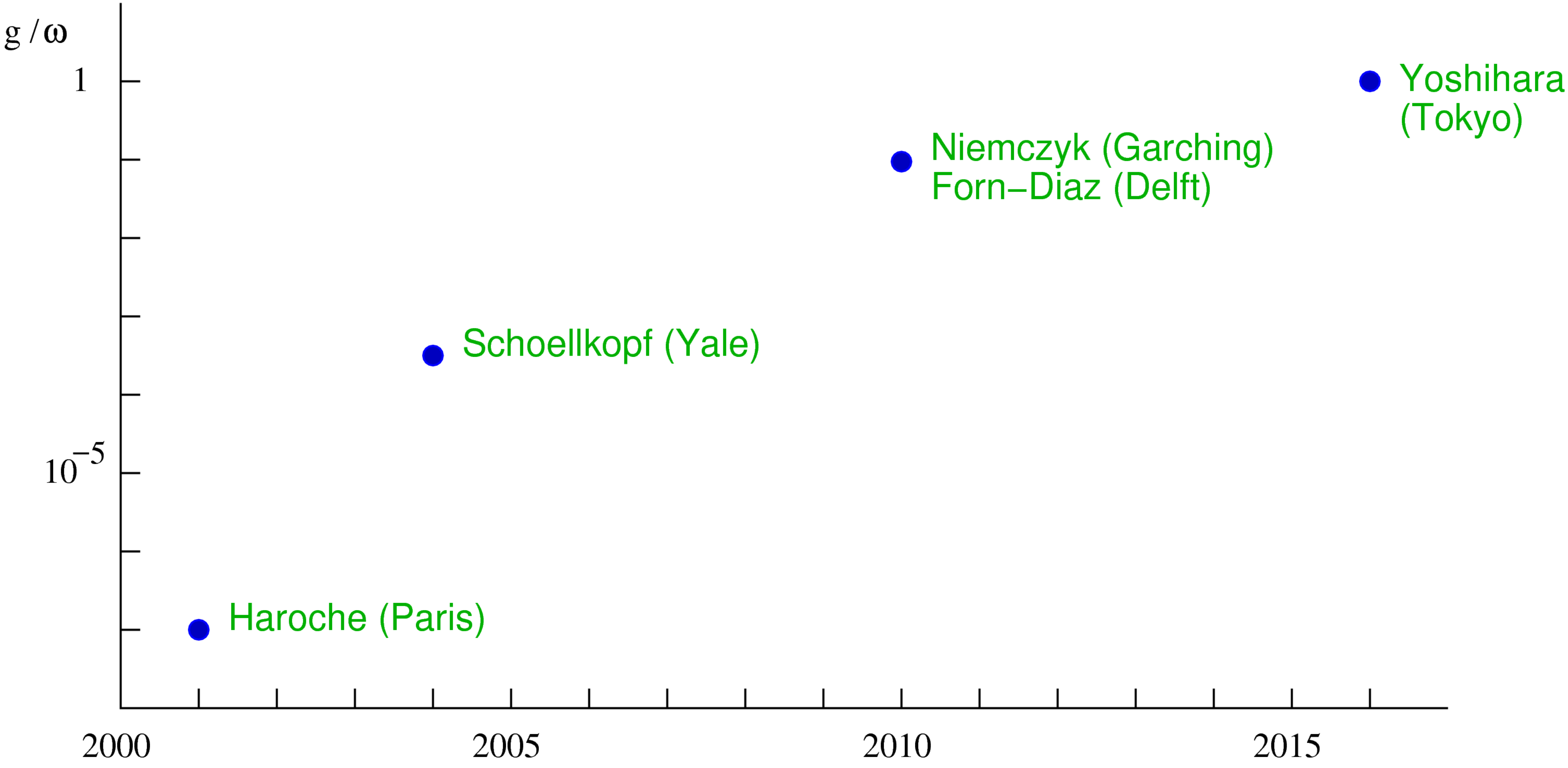
Wallraff *et al.* 2004

Blais *et al.* 2004

early quantum-optical applications (natural atoms)

$$H_R = \Delta\sigma_z + \omega a^\dagger a + g\sigma_x(a^\dagger + a)$$

relevant regime: $2\Delta \approx \omega, \quad g \ll \omega$



dramatic enhancement of coupling strength within cavity and circuit QED !

$$H_R = \omega a^\dagger a + \Delta\sigma_z + g(a^\dagger\sigma^- + a\sigma^+) + g(a^\dagger\sigma^+ + a\sigma^-)$$

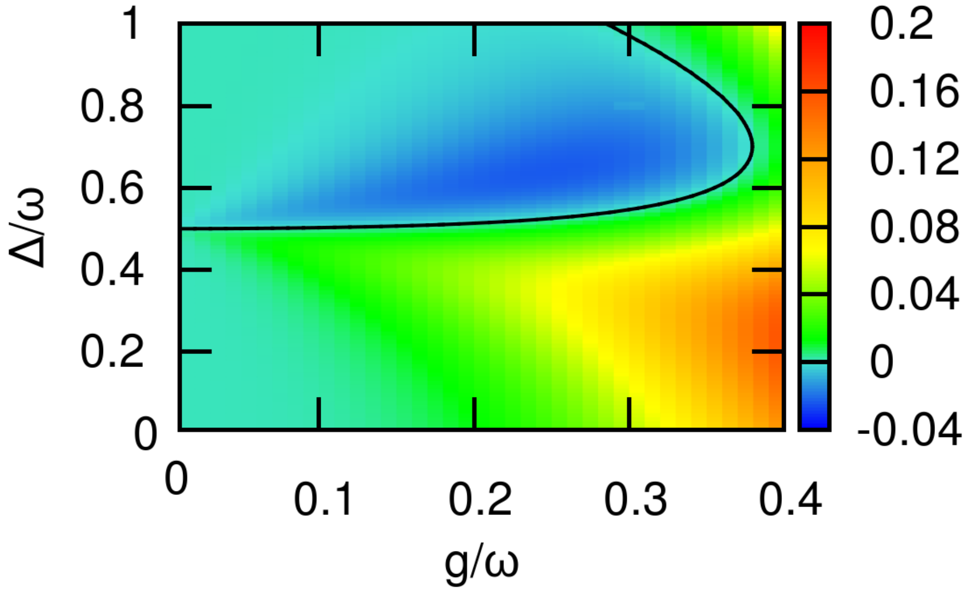
"rotating" terms

"counter-rotating" terms

$$H_{JC} = \omega a^\dagger a + \Delta\sigma_z + g(a^\dagger\sigma^- + a\sigma^+)$$

Jaynes, Cummings 1963

very good approximation for $2\Delta \approx \omega, \quad g \ll \omega$



time-averaged photon production

F.A.Wolf *et al.* Phys.Rev. A **87**, 023835 (2013) symmetry of the JC-model:

$V(\phi)^\dagger H_{JC} V(\phi) = H_{JC}$ $\qquad$ $V(\phi) = \exp i\phi[a^\dagger a + \sigma^+\sigma^-]$

$0 \leq \phi < 2\pi$

generator: $\hat{C} = a^\dagger a + \sigma^+\sigma^-$

group: $U(1)$ continuous

infinite many irreducible representations

→ infinite many invariant sub-spaces $\quad \dim \mathcal{H}_n = 2$

quantum Rabi model:

$$H_R = \omega a^\dagger a + \Delta\sigma_z + g\sigma_x(a + a^\dagger)$$

$V(\pi)^\dagger H_R V(\pi) = H_R$ $\qquad$ $V(\pi) = -\hat{P} = -\sigma_z(-1)^{a^\dagger a}$

$$\hat{P}|\pm 1, \phi(x)\rangle = \pm|\pm 1, \phi(-x)\rangle$$

group: $\mathbb{Z}_2$ discrete

two irreducible representations

→ two invariant sub-spaces $\quad \dim \mathcal{H}_\pm = \infty$

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1 continuous d.o.f.
1 discrete d.o.f.

Jaynes-Cummings model

$$H_{JC} = \omega a^\dagger a + \Delta \sigma_z + g(\sigma^+ a + \sigma^- a^\dagger)$$

$U(1)$ -symmetry

$$\psi_n^j = b_{+,n}^j |+, n-1\rangle + b_{-,n}^j |-, n\rangle$$

elimination of one continuous d.o.f.

$$\psi_n^j = |j, n\rangle, \quad j = 1, 2$$

1 continuous d.o.f.
1 discrete d.o.f.

quantum Rabi model

$$H_R = \omega a^\dagger a + \Delta \sigma_z + g \sigma_x (a + a^\dagger)$$

$\mathbb{Z}_2$ -symmetry

$$\psi_\pm^n = |+, \phi_\pm^n(x)\rangle \pm |-, \phi_\pm^n(-x)\rangle$$

elimination of one discrete d.o.f.

$$\psi_\pm^n = |\pm 1, n\rangle, \quad n = 0, 1, 2, \ldots$$

value of the symmetry operator

models with f_d discrete (quantum) and f_c continuous (classical) degrees of freedom:

Criterion on Quantum Integrability

If each eigenstate can be uniquely labeled by $f_d + f_c$ quantum numbers, the system is quantum integrable.
"fine-grained" description:

$$|\psi\rangle = |n_1, \ldots, n_{f_d}, m_1, \ldots, m_{f_c}\rangle$$

If each eigenstate can be uniquely labeled by a single quantum number (energy) for all values of the parameters, the system is not (quantum) integrable.

$$|\psi\rangle = |n\rangle$$

DB, *Phys.Rev.Lett.* **107,** 100401 (2011)

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Hydrogen atom:

$$\frac{\mathrm{d}^2\chi_l(r)}{\mathrm{d}r^2} + \left(2E + \frac{2e^2}{r} - \frac{l(l+1)}{r^2}\right)\chi_l(r) = 0$$

polynomial ansatz for normalizable wave functions

$$\chi_{l,n}(r) = r^{l+1}e^{-ar}P_n(r) \longrightarrow \qquad E_n = -\frac{1}{2n^2}$$

quantum Rabi model:

$$-\frac{1}{2}\frac{\mathrm{d}^2\phi_\pm(x)}{\mathrm{d}x^2} + \frac{\omega^2}{2}x^2\phi_\pm(x) + \sqrt{2\omega}gx\phi_\pm(x) \pm \Delta\phi_\pm(-x) = E_\pm\phi_\pm(x)$$

no polynomial ansatz possible except for special values of $g, \ \Delta$

(degenerate exceptional spectrum)

Bargmann space of analytic functions

Bargmann 1961

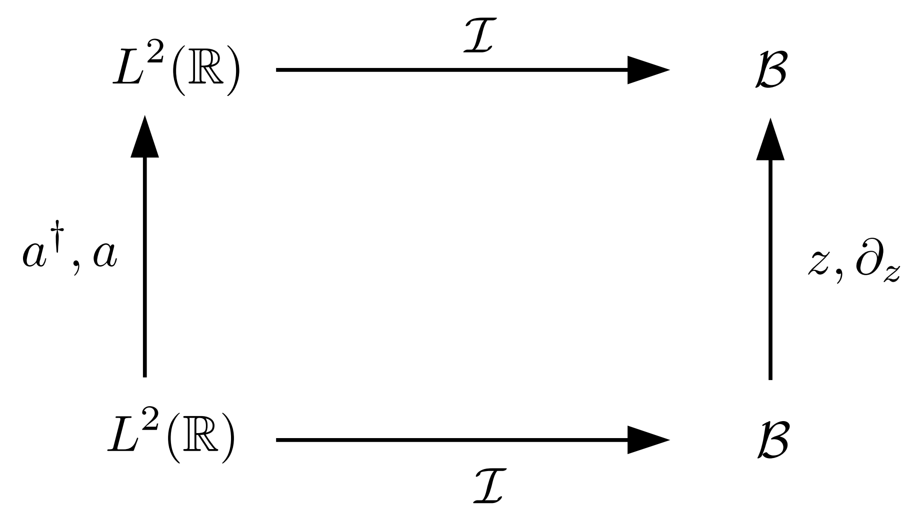
$$f(z,\bar z) \in \mathcal{B} \qquad \Longleftrightarrow \qquad \partial_{\bar z} f(z,\bar z) = 0, \quad \langle f|f\rangle < \infty$$

scalar product:

$$\langle f|g\rangle = \frac{1}{\pi}\int \mathrm{d}z\mathrm{d}\bar z e^{-|z|^2}\bar f(\bar z) g(z)$$

z is adjoint to ∂_z under the scalar product $\langle f|g\rangle$

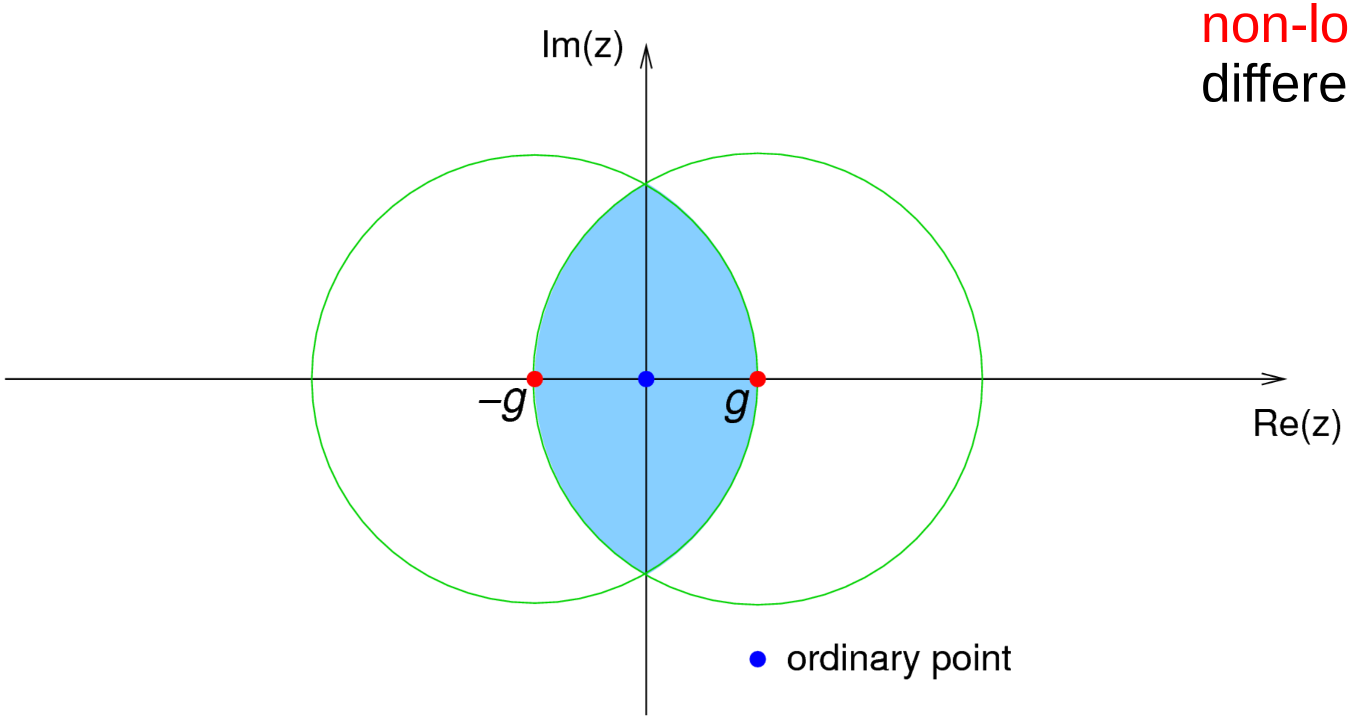
→ isometry between $L^2(\mathbb{R})$ and $\mathcal{B}$



quantum Rabi model: eigenvalue equation has regular singular points

$$z\frac{\mathrm{d}}{\mathrm{d}z}\phi_\pm(z) + g\left(\frac{\mathrm{d}}{\mathrm{d}z} + z\right)\phi_\pm(z) = E_\pm\phi_\pm(z) \mp \Delta\phi_\pm(-z)$$

non-local confluent Fuchsian differential equation



→ analyticity of wave functions yield spectral condition

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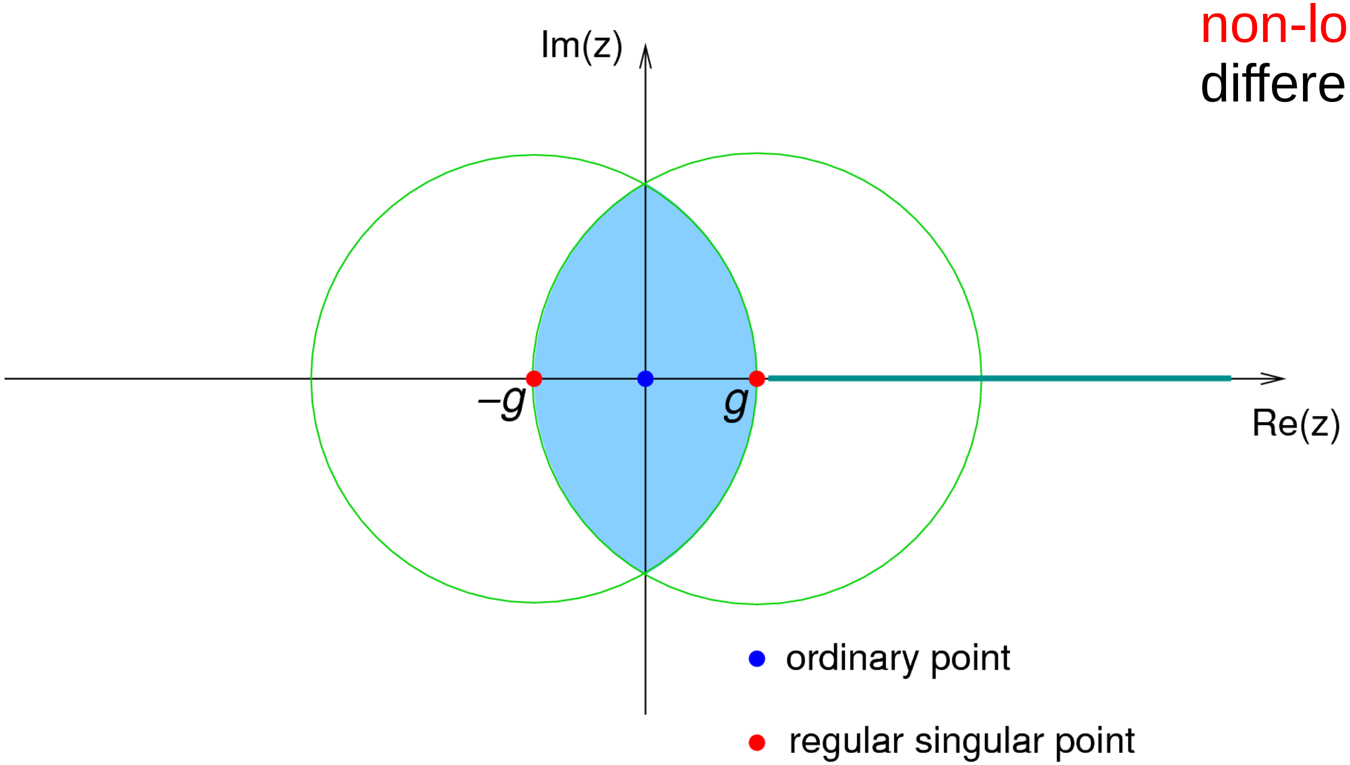

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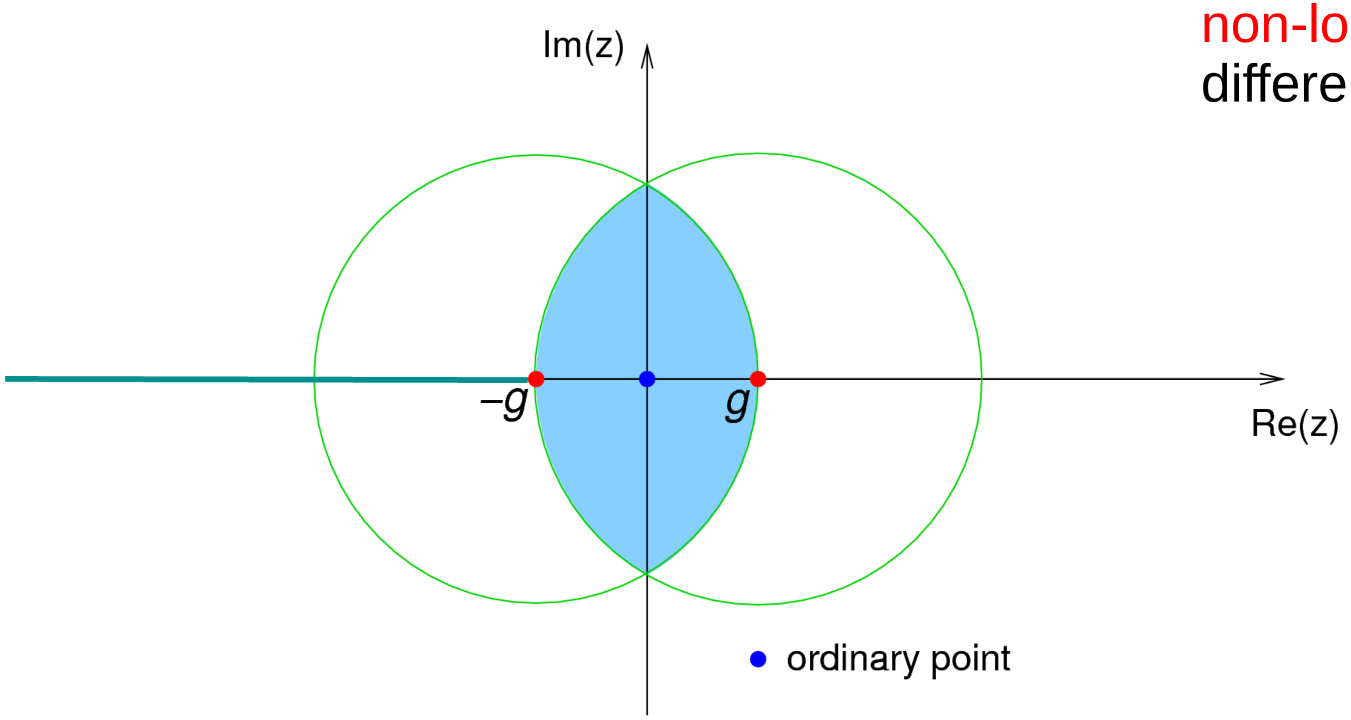

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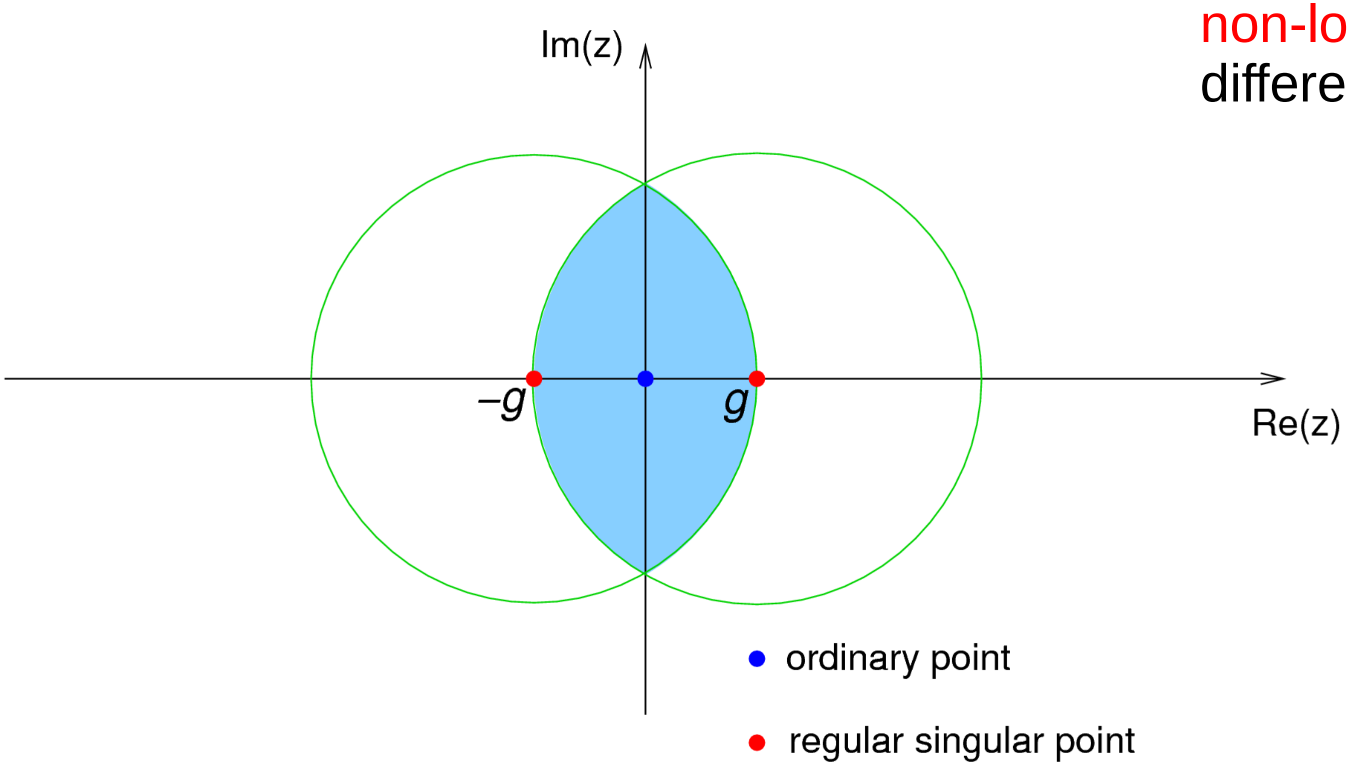
non-local confluent Fuchsian differential equation

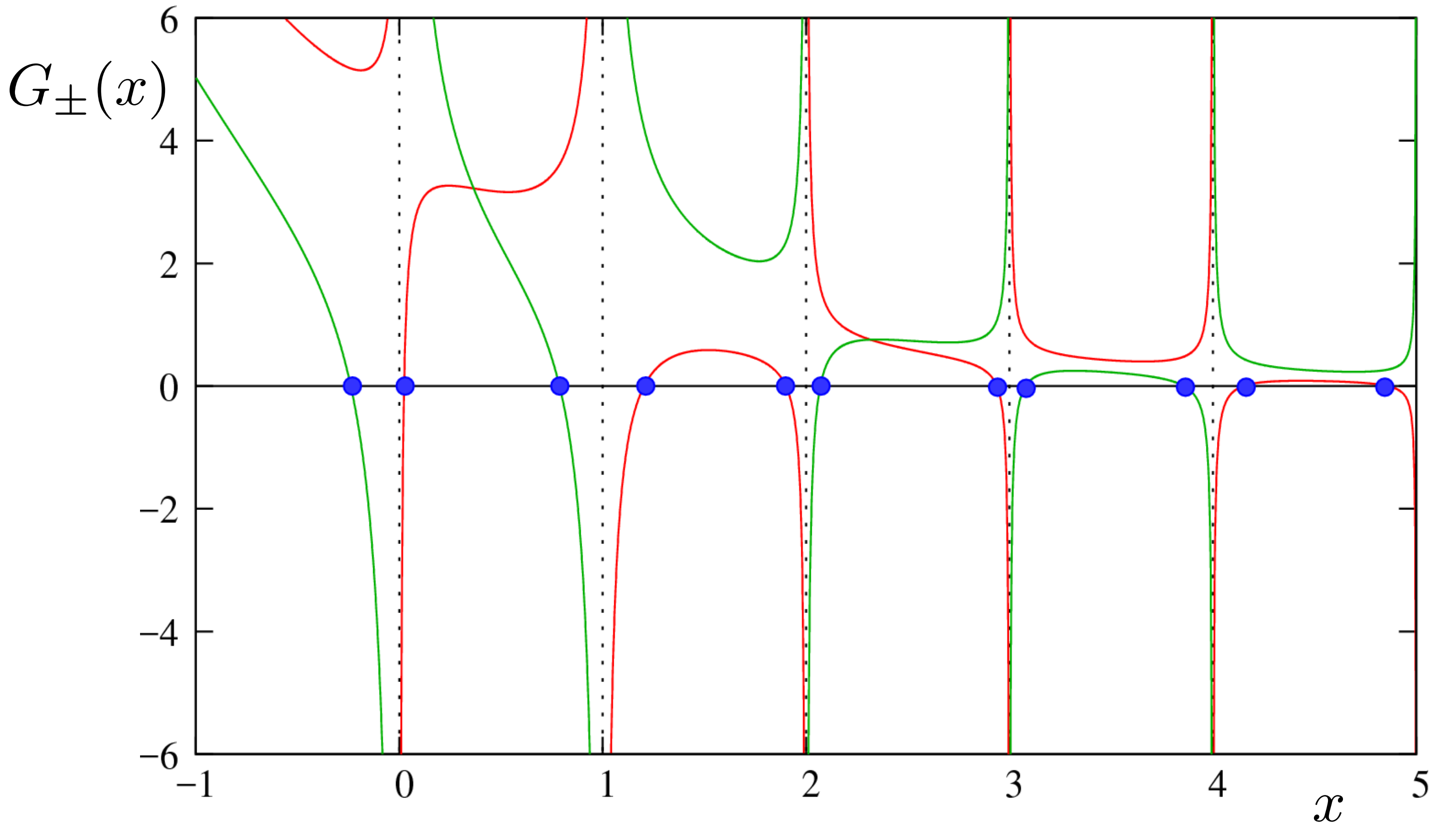


→ analyticity of wave functions yield spectral condition

$$G_\pm(x) = \left(1 \mp \frac{\Delta}{x}\right) H_c(\alpha,\gamma,\delta,p,\sigma;1/2) - \frac{1}{2x} H_c'(\alpha,\gamma,\delta,p,\sigma;1/2)$$

confluent Heun function   Ronveaux, Arscott 1995

$$E_n^\pm = x_n^\pm - g^2 \in \mathrm{spec}(H_\pm) \longrightarrow G_\pm(x_n^\pm) = 0$$



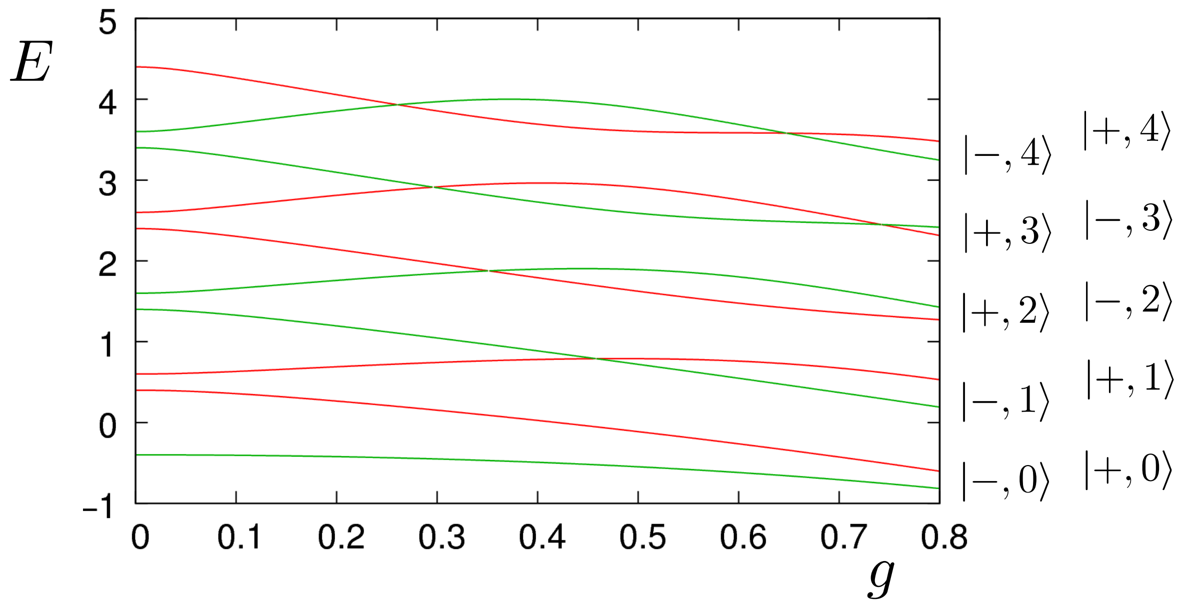
$g = \omega = 1$

$\Delta = 0.7$

DB, *Phys.Rev.Lett.* **107,** 100401 (2011)   DB, *J. Phys. B* **46**, 224007 (2013)

$f_c = 1 \quad f_d = 1 \quad \dim \mathcal{H}_d = 2$ # irreps of $\mathbb{Z}_2 = 2$

→ quantum Rabi model is integrable and solvable



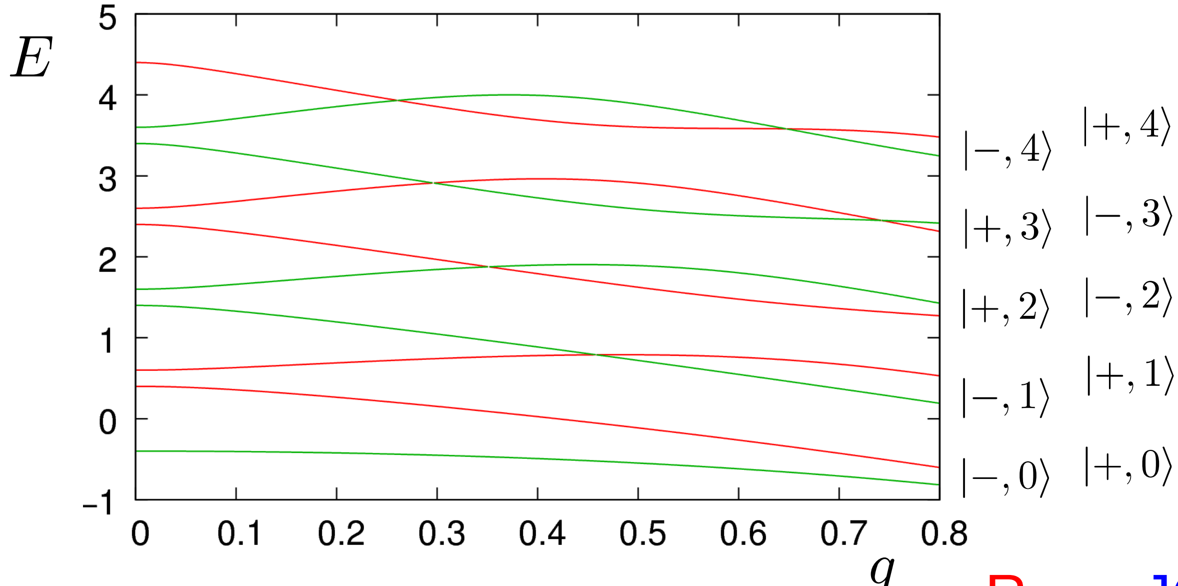
even and odd spectra of the Rabi model

quantum Rabi model is not integrable in the sense of Yang-Baxter

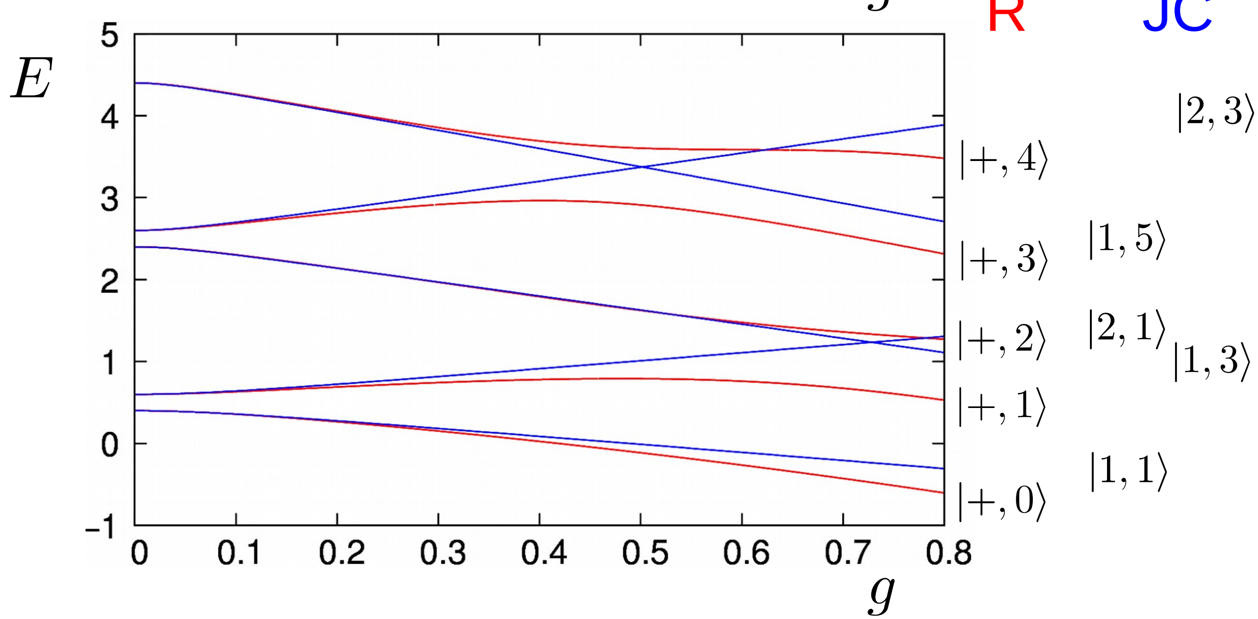
Amico *et al.* 2010
Batchelor, Zhou 2015

$f_c = 1 \quad f_d = 1 \quad \dim \mathcal{H}_d = 2$ # irreps of $\mathbb{Z}_2 = 2$

→ quantum Rabi model is integrable and solvable



even and odd spectra of the Rabi model



even Rabi spectrum
JC-spectrum for

$C = 1, 3, 5$

→ JC model is superintegrable

Miller *et al.* 2013

	f_c	f_d	$\dim \mathcal{H}_d$	symmetry	integrable
quantum Rabi model	1	1	2	$\mathbb{Z}_2$	yes
asymmetric QRM	1	1	2		no
two-photon QRM	1	1	2	$\mathbb{Z}_4$	yes*
Dicke model	1	N	2^N	$\mathbb{Z}_2$	no
Jaynes-Cummings model	1	1	2	$U(1)$	yes*

* superintegrable

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two-photon quantum Rabi model

$$H_{2p} = \omega a^\dagger a + g(a^{\dagger 2} + a^2)\sigma_x + \Delta\sigma_z$$

coupling is non-linear in the bosonic operators

same degrees of freedom as in QRM but larger discrete symmetry

$$\hat{P}_1 = e^{i\pi a^\dagger a} \qquad \hat{P}_2 = e^{i\frac{\pi}{2}a^\dagger a} \otimes \sigma_z \qquad \hat{P}_1^2 = 1\!\!1, \quad \hat{P}_2^2 = \hat{P}_1$$

$$[H_{2p}, \hat{P}_1] = [H_{2p}, \hat{P}_2] = 0$$

⟶ $\hat{P}_2$ generates $\mathbb{Z}_4$-symmetry of H_{2p}

⟶ H_{2p} is integrable

in $\mathcal{B}$: $\hat{P}_1[\phi](z) = \phi(-z)$ $\hat{P}_2 = \hat{T} \otimes \sigma_z$ $\hat{T}[\phi](z) = \phi(iz)$

even and odd functions in $\mathcal{B}$ $\mathcal{B}_\pm = \{\phi(z) | \phi(z) = \pm\phi(-z)\}$

$\mathbb{Z}_4$-symmetry leads to four invariant subspaces

$$\mathcal{H} = \mathcal{H}_+^+ \oplus \mathcal{H}_+^- \oplus \mathcal{H}_-^+ \oplus \mathcal{H}_-^-$$

$\mathcal{H}_+^\pm$ and $\mathcal{H}_-^\pm$ are isomorphic to $\mathcal{B}_+$ and $\mathcal{B}_-$

eigenvalue equation in $\mathcal{H}_+^+$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} + \omega z \frac{\mathrm{d}}{\mathrm{d}z} + z^2 - E\right] \psi(z^2) + \Delta\psi(-z^2) = 0 \qquad (g = 1)$$

non-local 2nd order ODE in $\mathcal{B}_+$ (even analytic functions) equivalent system

$$\phi_1'' + \omega z \phi_1' + (z^2 - E)\phi_1 = -\Delta \phi_2$$

$$\phi_2'' - \omega z \phi_2' + (z^2 + E)\phi_2 = \Delta \phi_1$$

$$\phi_2(z) = \phi_1(iz)$$

in contrast to QRM, no singular points except at $z = \infty$

single irregular singular point has s-rank 3

Slavyanov, Lay 2000

→ asymptotic behavior of normal solutions

$$\psi(z) = e^{\frac{\gamma}{2} z^2 + \alpha z} z^{\rho} (c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots)$$

only normalizable if $|\gamma| < 1$

plane waves in $\mathcal{B}$

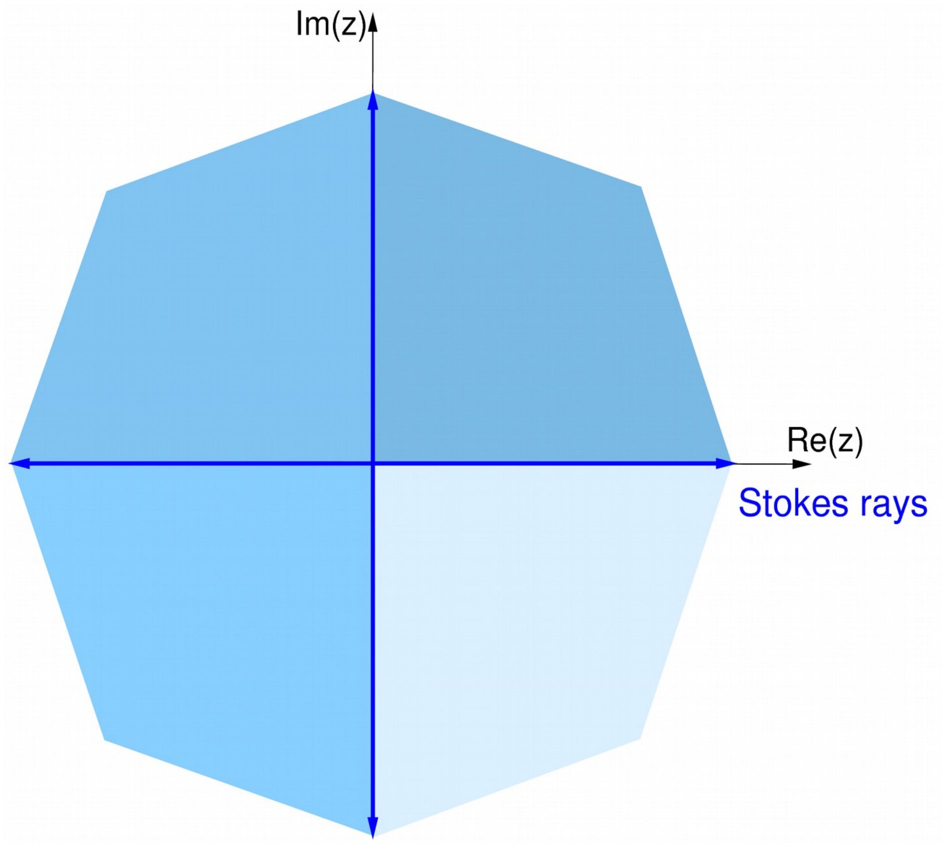
$$|p\rangle = e^{ipx} \xrightarrow{\mathcal{I}} f_p(z) = \frac{e^{-p^2/2}}{\pi^{1/4}} e^{\frac{1}{2} z^2 + i\sqrt{2} p z}$$

asymptotic behavior of normal solutions

$$\psi(z) = e^{\frac{\gamma}{2}z^2 + \alpha z} z^{\rho} (c_0 + c_1 z^{-1} + c_2 z^{-2} + \dots)$$

Stokes phenomenon:

expansion with fixed γ, α, ρ only valid for single Stokes sector



$$\gamma = \pm \left[\frac{\omega}{2} \pm \sqrt{\frac{\omega^2}{4} - 1} \right]$$

$$\omega \leq 2 \rightarrow |\gamma| = 1$$

⟶

states only normalizable for

$$\omega > 2$$

asymptotic behavior of normal solutions

$$\psi(z) = e^{\frac{\gamma}{2}z^2+\alpha z} z^\rho (c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots)$$

Stokes phenomenon:

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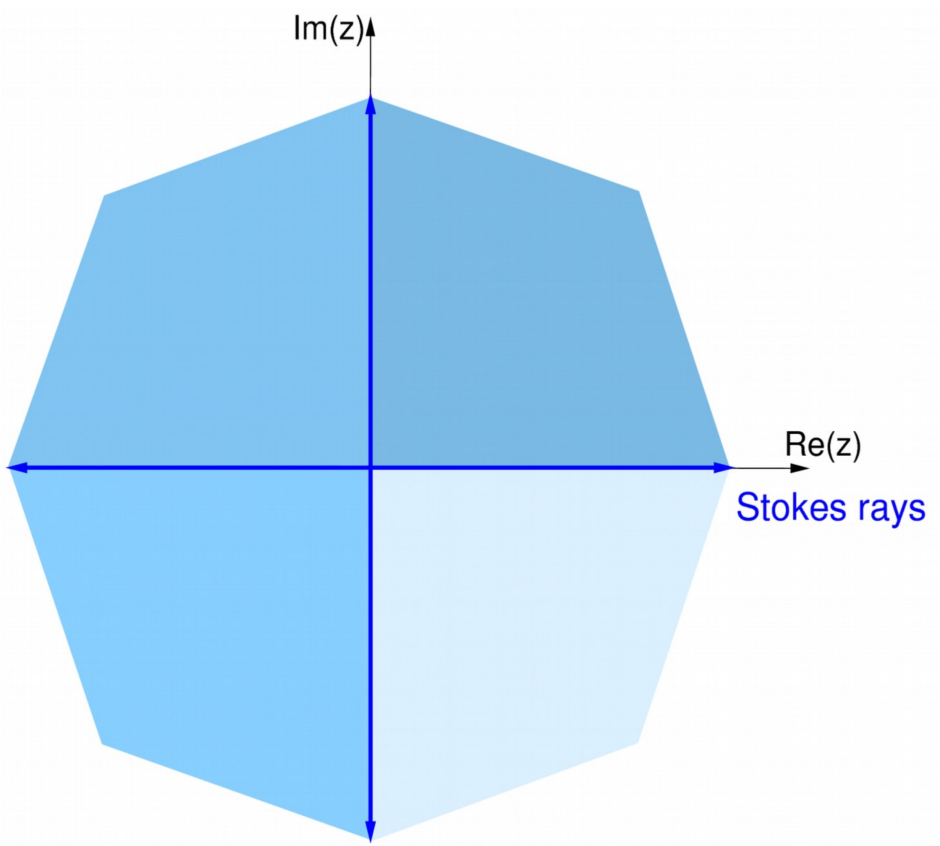


$\omega > 2:$

$$\gamma_{1,2} = \pm\left(\omega/2 - \sqrt{\omega^2/4 - 1}\right)$$

$|\gamma_{1,2}| < 1$ admissible

$$\gamma_{3,4} = \pm\left(\omega/2 + \sqrt{\omega^2/4 - 1}\right)$$

$|\gamma_{3,4}| > 1$ not admissible

how to find the solution containing only components with $\gamma_{1,2}$?

scale transformation in $L^2(\mathbb{R})$

$$I_\theta[\phi](x) = \phi(e^\theta x) \qquad -\infty < \theta < \infty$$

I_θ not unitary: $\langle I_\theta[\phi] | I_\theta[\psi] \rangle = e^{-\theta} \langle \phi | \psi \rangle$

$$a = \frac{1}{\sqrt{2}}(x + \partial_x) \quad \rightarrow \quad \mathrm{ch}(\theta) a + \mathrm{sh}(\theta) a^\dagger$$

$$a^\dagger = \frac{1}{\sqrt{2}}(x - \partial_x) \quad \rightarrow \quad \mathrm{ch}(\theta) a^\dagger + \mathrm{sh}(\theta) a$$

bosonic Bogoliubov (squeezing) transformation in $\mathcal{B}$

$$I_\theta = e^{-\theta/2} \exp \frac{\theta}{2}(\partial_z^2 - z^2)$$

transformation of two-photon problem with $\mathrm{th}(\theta) = -\frac{2}{\omega}$

$$\omega_1 z \phi_1' - E_1 \phi_1 = -\Delta \phi_2$$

$$2\mathrm{ch}(2\theta)\phi_2'' + \omega_2 z \phi_2' + [2\mathrm{ch}(2\theta) z^2 + E_2]\phi_2 = \Delta \phi_1$$

$$\omega_1 = \mathrm{sh}(2|\theta|)(\omega^2/2 - 2)$$

regular singular point at $z = 0$ besides irregular singular point at ∞

s-rank at ∞ is still 3: $\phi_1(z) = e^{\frac{\gamma}{2} z^2 + \alpha z} z^\rho (c_0 + c_1 z^{-1} + \ldots)$

$\gamma_1 = \frac{2}{\omega} < 1$ admissible

$\gamma_2 = \frac{\omega}{2} > 1$ not admissible apply $\mathbb{Z}_4$-symmetry $\qquad \hat{T} = \exp i\frac{\pi}{2} z\partial_z$

$\phi_2(z) = \hat{T}_\theta[\phi_1](z) \qquad \hat{T}_\theta = I_\theta \hat{T} I_\theta^{-1}$

$$I_\theta = e^{-\theta/2} \exp\frac{\theta}{2}(\partial_z^2 - z^2)$$

$z\partial_z - \mathbb{1}/2, \quad z^2/2, \quad \partial_z^2/2$ furnish representation of $\mathfrak{sl}(2,\mathbb{R})$

→ computation in defining representation of $SL(2,\mathbb{R})$

$$\phi_1(z) = A_1 \exp\frac{z^2}{\omega} + B_1 \exp\frac{\omega z^2}{4} + \dots \qquad \hat{T}_\theta\left[\exp\gamma\frac{z^2}{2}\right](z) = \exp\eta(\gamma)\frac{z^2}{2}$$

$$\phi_2(z) = A_2 \exp\frac{z^2}{\omega} + B_2 \exp\frac{\omega z^2}{4} + \dots \qquad \eta(\gamma) = -\frac{\gamma + \mathrm{th}(2\theta)}{\mathrm{th}(2\theta)\gamma + 1}$$

$$\gamma_1 = \frac{2}{\omega} < 1 \quad \rightarrow \quad \eta(\gamma_1) = 0$$

$$\gamma_2 = \frac{\omega}{2} > 1 \quad \rightarrow \quad \eta(\gamma_2) = \infty$$

⟶

only normalizable functions are mapped by $\hat{T}_\theta$ onto functions with asymptotics allowed by the ODE

⟶ spectral condition $\phi_2(z) = \hat{T}_\theta[\phi_1](z)$

may be evaluated using local Frobenius expansion around $z = 0$

$$\phi_2(z) = \sum_{n=0}^{\infty} a_n(E) z^{2n}$$

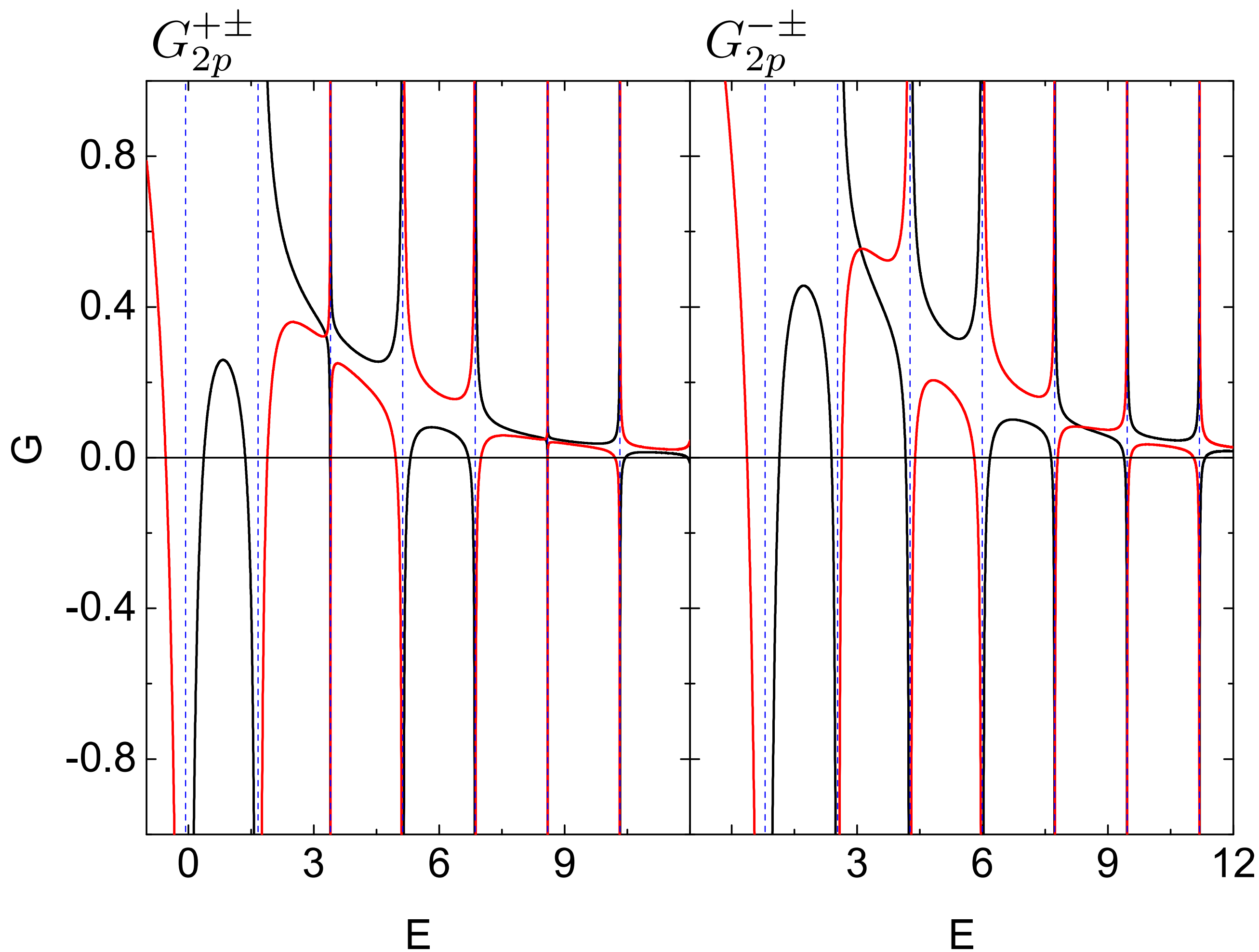
$$a_0 = 1$$

$$G_{2p}^{++}(E) = 1 - \sum_{n=0}^{\infty} \frac{\Delta}{E_1(E) - 2n\omega_1} a_n(E) \frac{(2n)!}{\omega^n n!}$$

$$G_{2p}^{++}(E) = 1 - \sum_{n=0}^{\infty} \frac{\Delta}{E_1(E) - 2n\omega_1} a_n(E) \frac{(2n)!}{\omega^n n!}$$

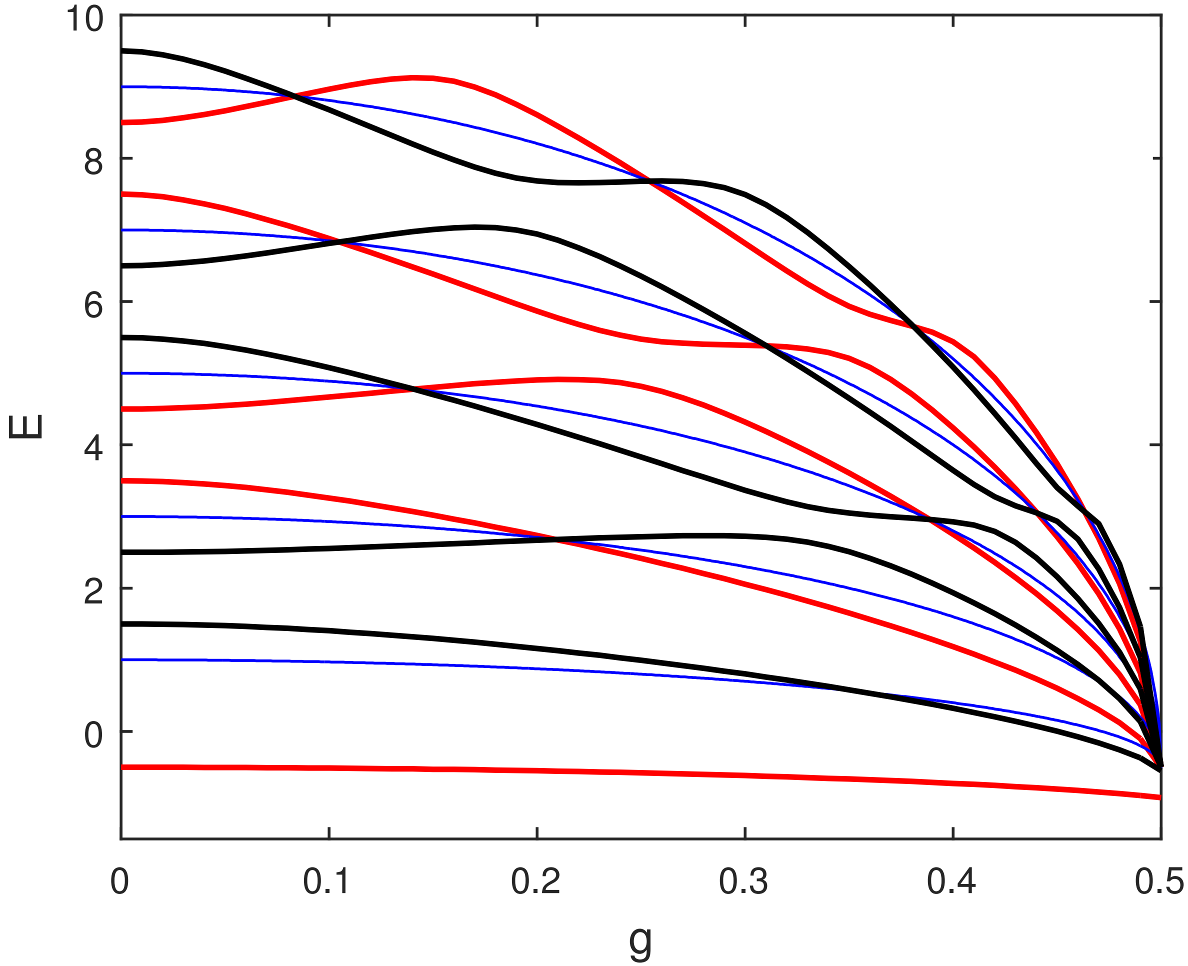
$$E_1(E) = E - \mathrm{sh}(2\theta) - \omega \mathrm{sh}^2(\theta)$$

distance between consecutive poles of $G_{2p}^{++}(E)$: $\quad 2\sqrt{\omega^2 - 4}$



$\Delta = 2$
$g = 1$
$\omega = 4$

→ spectral collapse if critical coupling $g = \omega/2$ is approached



QH Chen *et al.* 2012

S Felicetti, JS Pedernales, IL Egusquiza, G Romero, L Lamata, DB and E Solano, Phys. Rev. A **92**, 033817 (2015)

LW Duan, YF Xie, DB and QH Chen, (2016) to be published in J. Phys. A

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anisotropic Dicke model

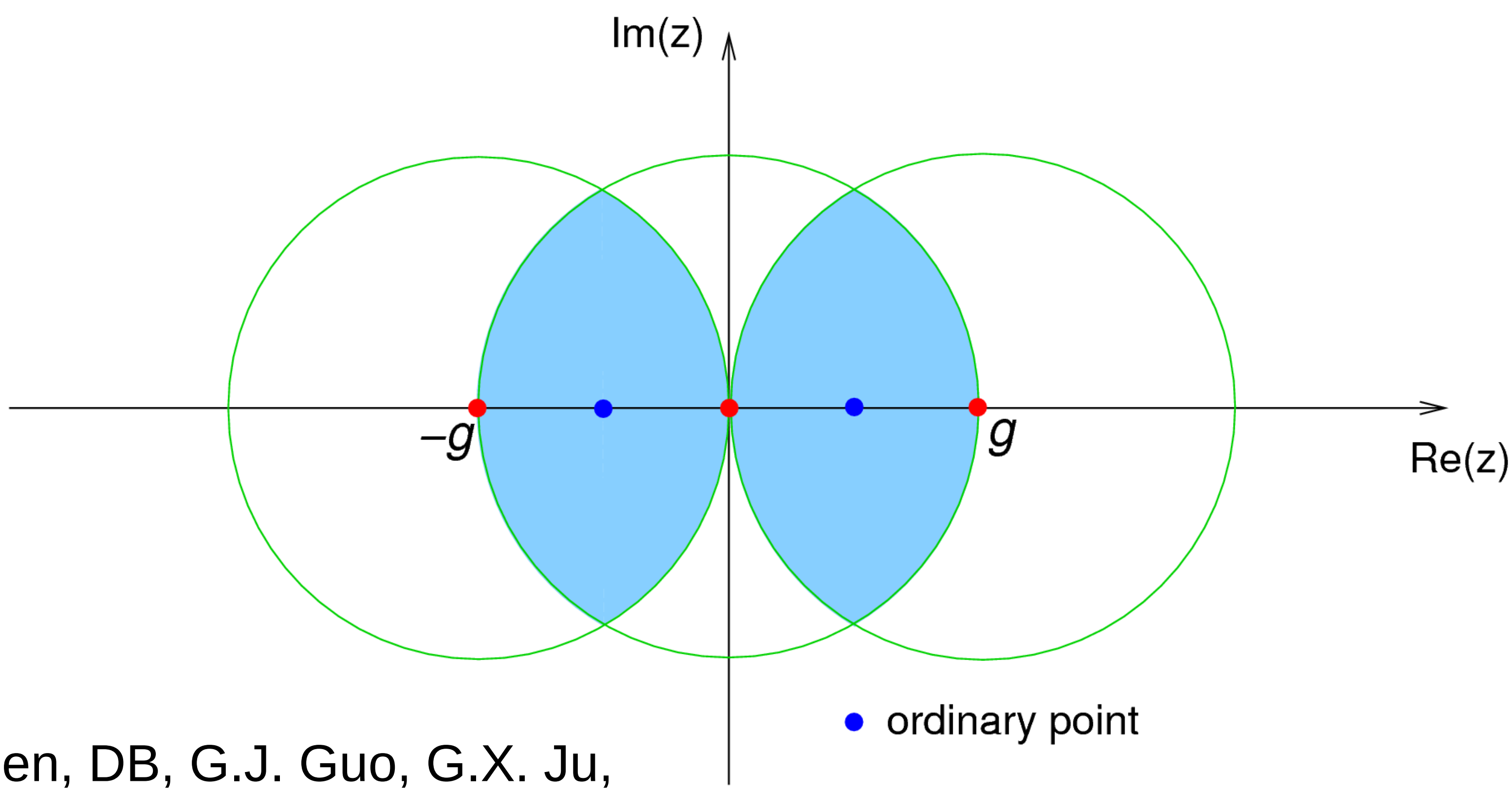
$$H_D = \omega a^\dagger a + \sum_{i=1}^{N} \Delta_i \sigma_i^z + \sum_{i=1}^{N} g_i (a + a^\dagger) \sigma_i^x$$

$\dim \mathcal{H}_d = 2^N$ ⟶ non-integrable for all $N \geq 2$

$N = 2:$

$g_1 = g_2$

$\Delta_1 \neq \Delta_2$



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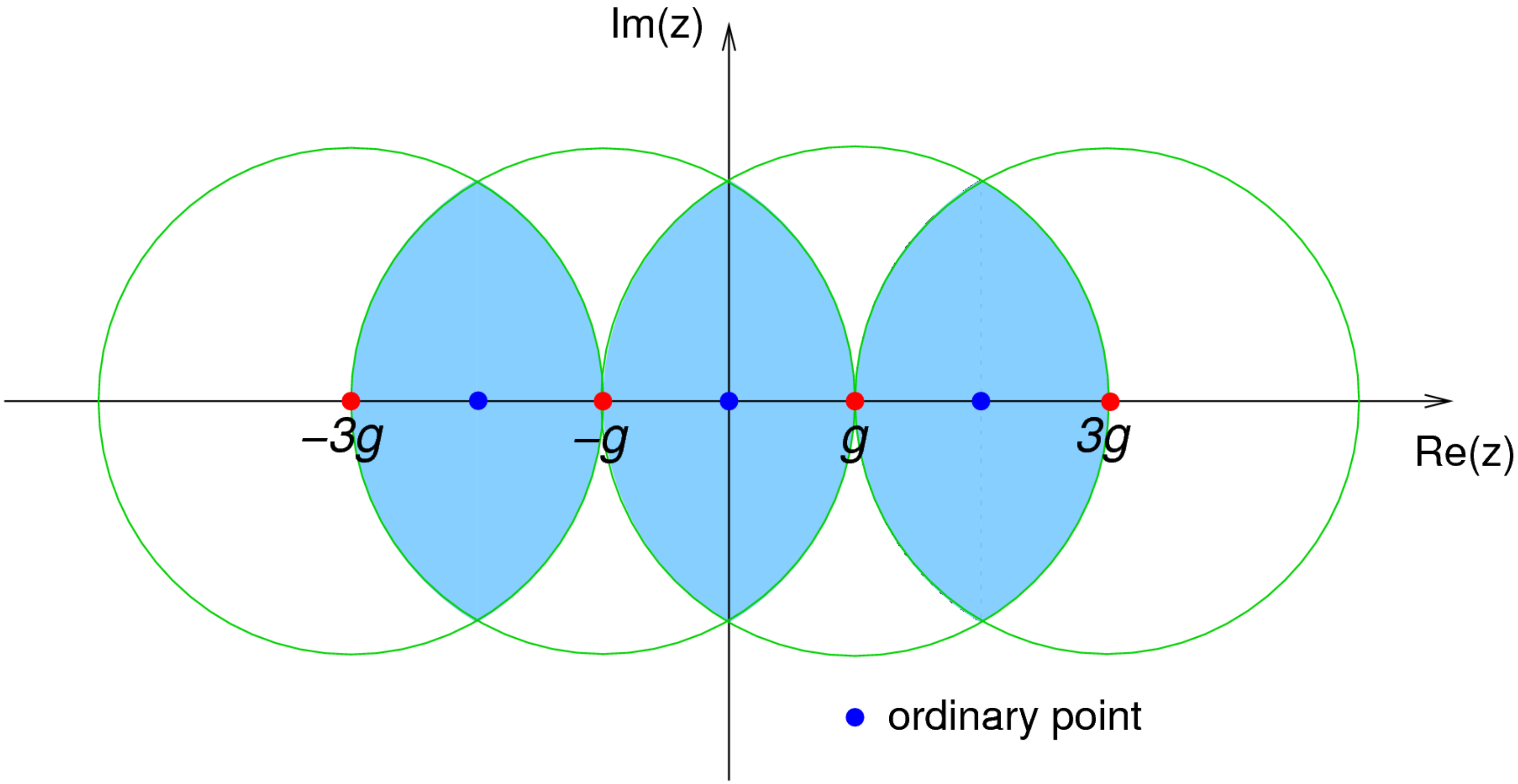
isotropic case for $N = 3$

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$$

$\dim \mathcal{H}_d = 4 \quad (\text{spin } 3/2)$

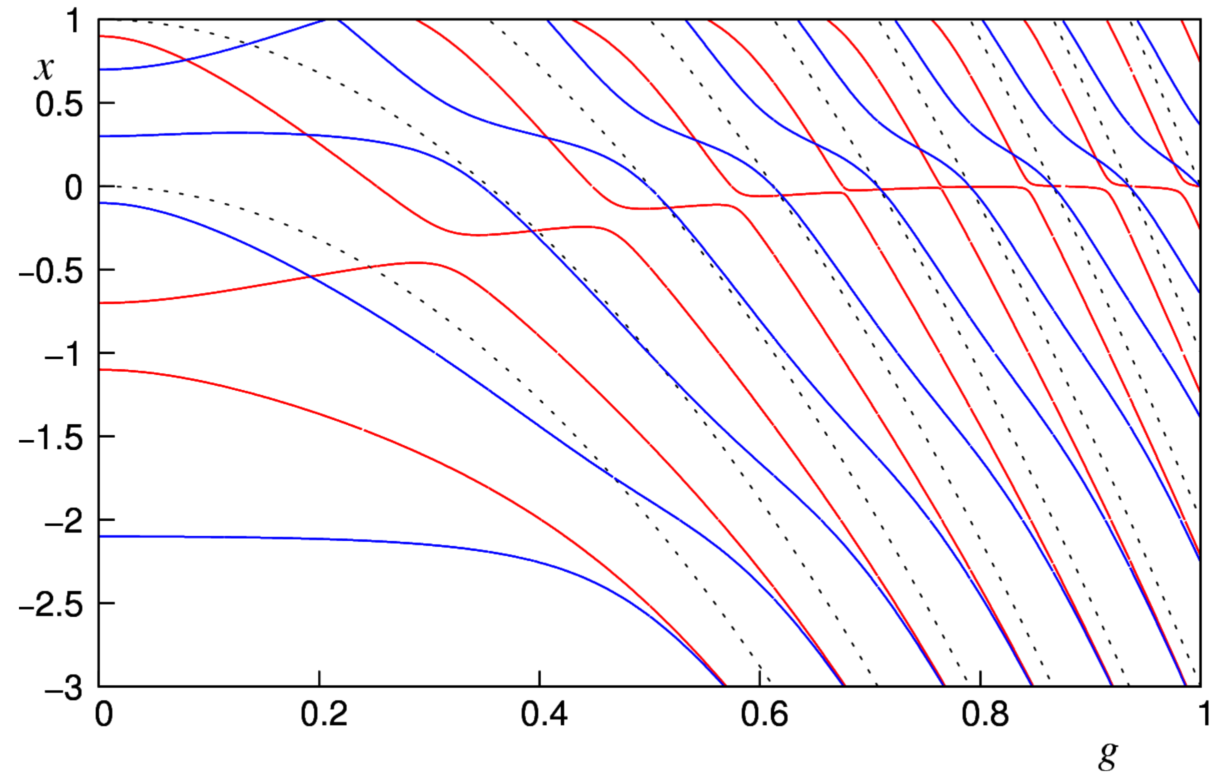


4 regular singular points and one s-rank 2 singular point at $z = \infty$

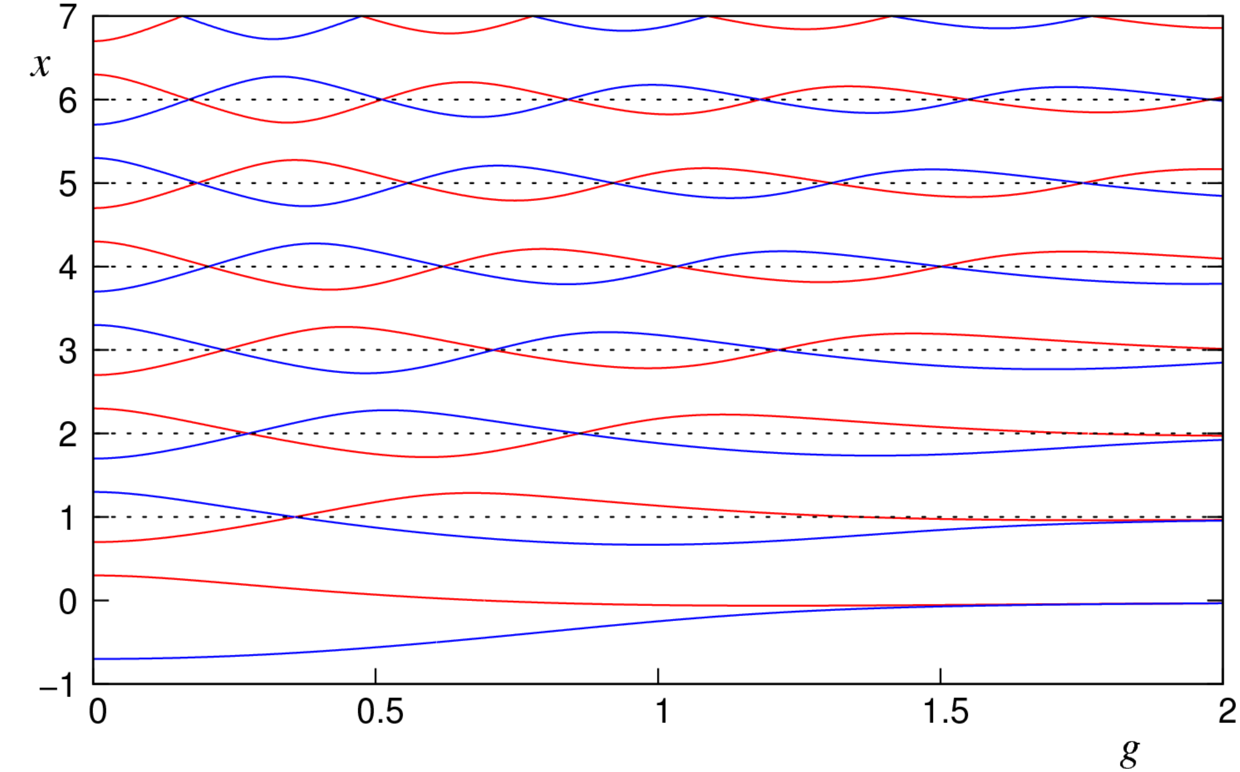
→ all formal solutions are normalizable

Dicke spectrum $N = 3$

$x = E + g^2$



Rabi spectrum



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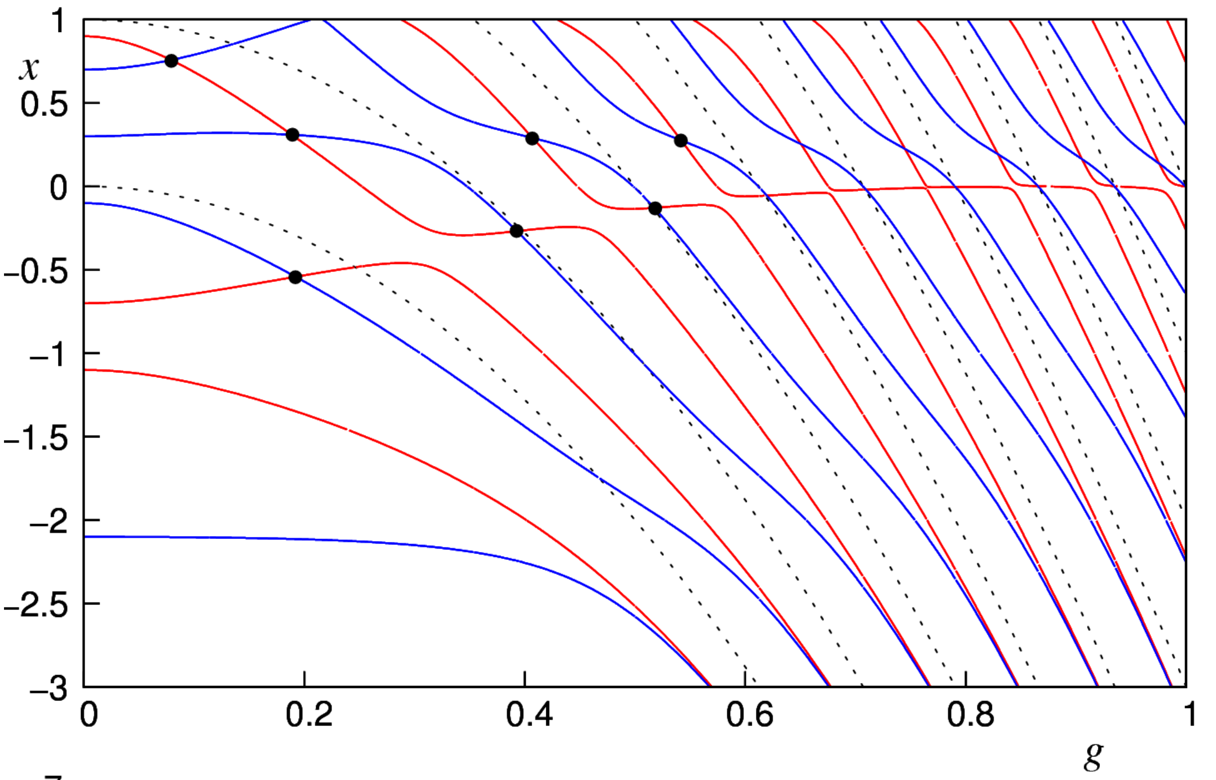
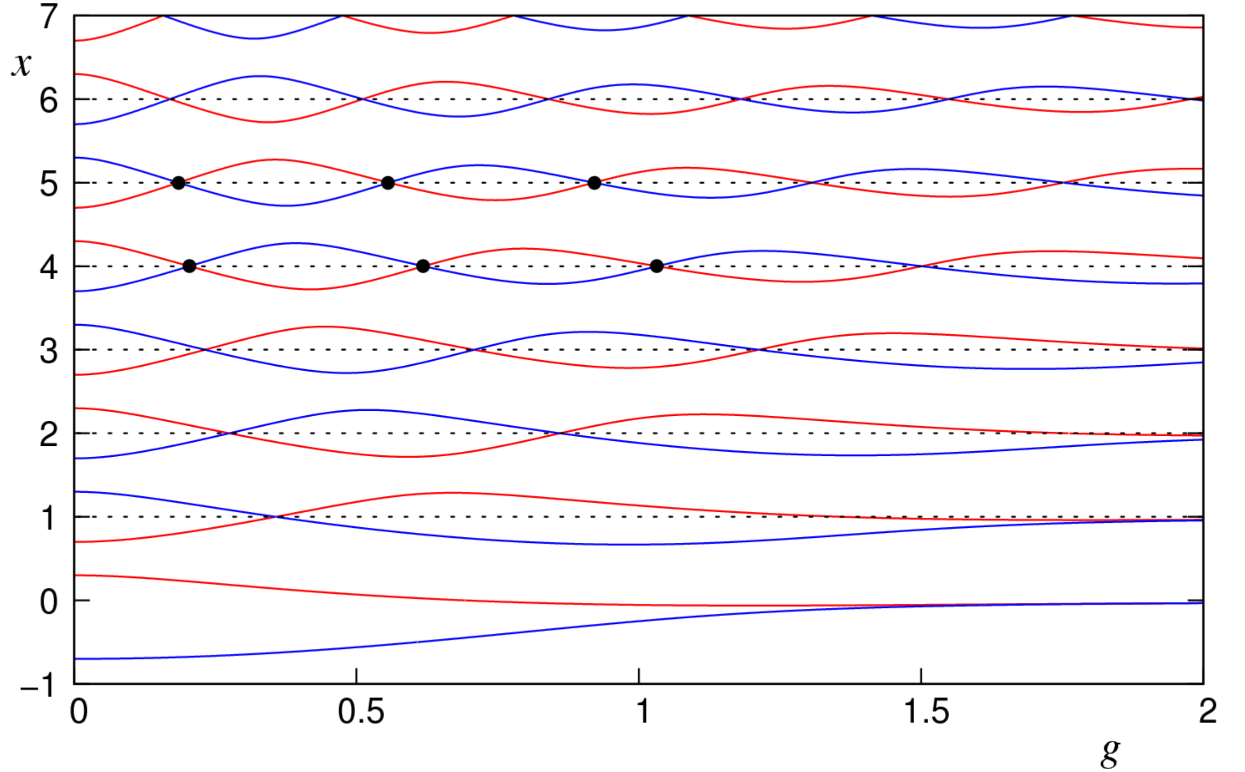
Dicke spectrum $N = 3$

$x = E + g^2$



Rabi spectrum



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conclusions

- criterion for quantum integrability for systems with few degrees of freedom
- discrete symmetry relates global and local properties of ODEs in $\mathbb{C}$ $\longrightarrow$
- derivation of G-function for two-photon QRM
- explanation of spectral collapse
- non-integrable Dicke model is solvable within G-function formalism

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Guillermo Romero, Fabio Vallone, Simone Felicetti, Julen Pedernales, Lucas Lamata, Inigo Egusquiza, Mikel Sanz, Enrique Solano	Bilbao
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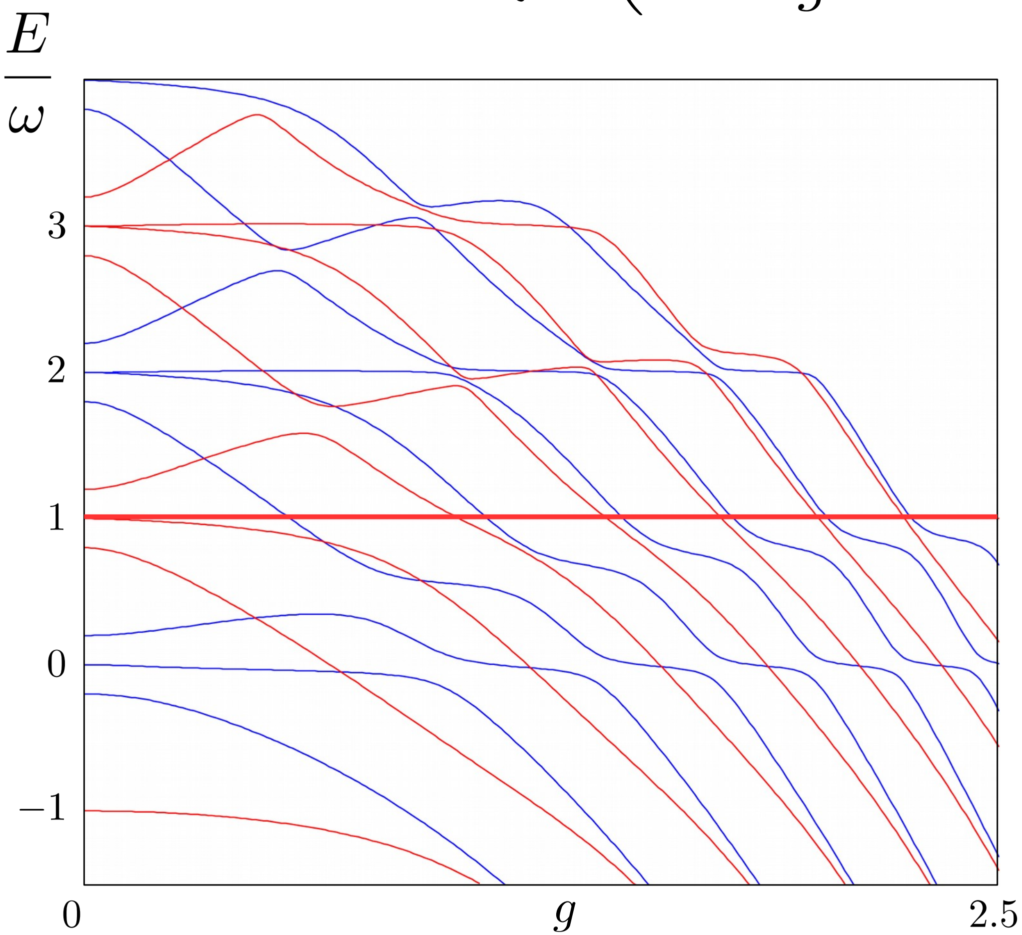
anisotropic Dicke model for $N = 2$

$$H_{D_2} = \omega a^\dagger a + g_1\sigma_{1x}(a + a^\dagger) + g_2\sigma_{2x}(a + a^\dagger) + \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z}$$

$$g_1 = g_2, \quad \Delta_1 + \Delta_2 = \omega$$

$$|\psi\rangle_e = \frac{1}{\mathcal{N}}\left(\frac{2(\Delta_1 - \Delta_2)}{g}|0,\uparrow,\uparrow\rangle - |1,\uparrow,\downarrow\rangle + |1,\downarrow,\uparrow\rangle\right)$$



$\longleftarrow$ $E = \omega$ independent of g !

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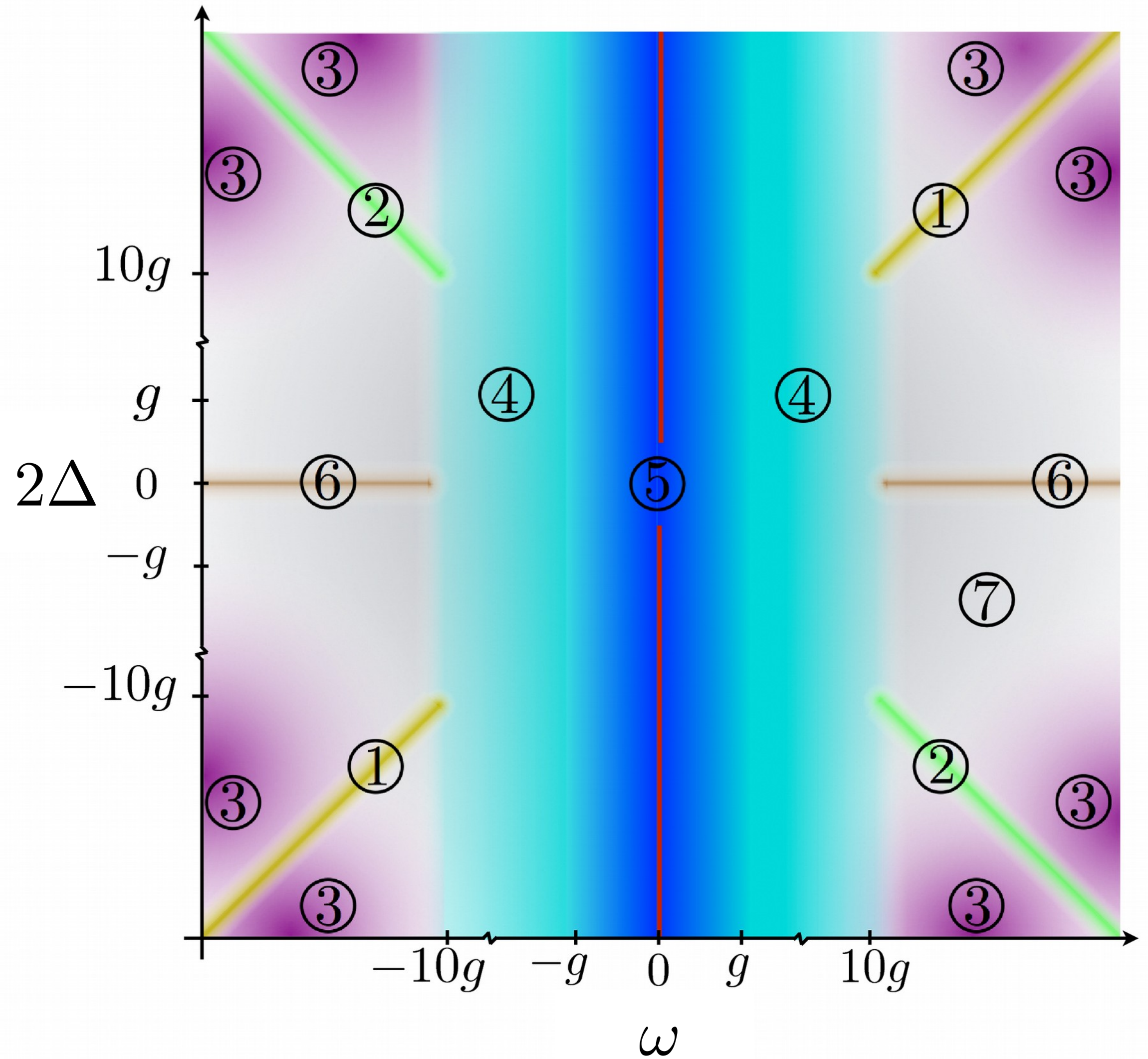

1 JC regime

2 anti-JC

3 dispersive

4 ultra-strong coupling

5 deep strong coupling

6 decoupling

7 dark zone

cover figure of the special issue of *Journal of Physics A*

“Semi-classical and quantum Rabi models”